
DESIRABILITY-BASED MULTI-RESPONSE OPTIMIZATION OF FLEXURAL PERFORMANCE IN BAMBOO AND STEEL REINFORCED CONCRETE BEAMS

Chukwu Mbazor^{1,*}, J. M. Kaura¹, A. Ocholi¹, J. Ochepo¹

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Email: chuksmb@gmail.com*, jmkaura@abu.edu.ng, amanaocholi@gmail.com, ochepo1@gmail.com

*Corresponding author: Chukwu Mbazor

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ABSTRACT

This study presents comparative optimization and predictive modeling of the flexural properties (deflection and failure load) of bamboo- and steel-reinforced concrete beams at a target grade of 25 MPa. Using Optimal Custom Design (OCD) in Response Surface Methodology (RSM), the effects of curing age (7–28 days) and reinforcement areas (145–580 mm² for bamboo; 50.3–201.2 mm² for steel) were analyzed. Sequential model sum of squares recommended third-order cubic polynomial models, while analysis of variance (ANOVA) validated their significance at a 99.99% confidence level. Residual diagnostics indicated approximately normally distributed experimental errors. The fit summaries showed high predictive power, with adjusted R² values of 0.9984–0.9992, predicted values agreeing within 0.2, and adequate precision ratios ranging from 175.18–242.13, exceeding the threshold of 4. Multi-response optimization minimized deflection and maximized failure load. For bamboo-reinforced concrete, the optimum was 9.588 days curing age and 580.000 mm² bamboo area, producing 4.692 mm deflection, 9.471 kN failure load, and 0.785 desirability. For steel-reinforced concrete, the optimum was 9.926 days and 201.200 mm² steel area, producing 6.229 mm deflection and 13.865 kN failure load. Approximately 10 days curing age was optimal for both. Steel-reinforced concrete was 24.67% more ductile and sustained 31.70% more load than bamboo-reinforced concrete. Desirability values were 0.785 and 0.782, respectively. The findings demonstrate bamboo's potential as a substitute for steel reinforcement in low-cost housing.

Keywords: Bamboo-reinforced concrete; Response Surface Methodology (RSM); Optimal Custom Design (OCD); Flexural properties (deflection & failure load); multi-response optimization; Desirability function

1. INTRODUCTION

Steel-reinforced concrete remains one of the most important developments in structural engineering since its introduction in the nineteenth century. Its widespread application is largely attributable to the favourable properties of concrete, including high compressive strength, durability, relatively low maintenance demand, and the ability to be moulded into different forms (McCormac & Brown, 2014). Steel complements concrete by addressing its principal limitation, namely, its relatively poor tensile capacity. The continued success of steel-concrete composite action is also associated with the comparatively low cost of steel among metallic materials. Iron, which constitutes the principal component of steel, is described as the least expensive and most extensively used metal (USGS, 2019). Fibre Reinforced Polymers (FRP) offer mechanical characteristics comparable to, and durability greater than, those of steel; however, their substantially higher cost limits their economic attractiveness (Burgoyne & Balafas, 2007).

A comparison based on volume is more appropriate when steel is evaluated against bamboo because the strength contribution of a reinforcing material depends on its cross-sectional area rather than simply on its mass. Steel is also considerably denser than bamboo, with reported densities of approximately 7.6 ton/m³ and 0.65 ton/m³, respectively. Although establishing an exact market price for bamboo may be difficult, available evidence indicates that bamboo is substantially less costly than steel. Its prevalence in developing countries further enhances its potential as a locally accessible reinforcement material, particularly in locations where steel may be difficult to obtain (Ghavami, 1995; Sharma, 1990).

The search for alternatives to steel has consequently been encouraged by the need for reinforcement materials that are economical and readily obtainable, particularly within developing economies. The concept of using bamboo to reinforce concrete is also longstanding. Chow (1914) conducted an early investigation at MIT into bamboo reinforcement, motivated by bamboo's high tensile strength and widespread availability in China. Since then, reduction in construction expenditure has remained a major justification for considering bamboo as reinforcement. At the same time, population growth continues to increase demand for construction materials, contributing to rising material costs and greater environmental pressure. The need for economical and environmentally sustainable construction materials is particularly significant because the construction sector accounts for about 50% of global raw-material consumption (Pacheco-Torgal & Labrincha, 2013).

Response Surface Methodology (RSM) comprises mathematical and statistical procedures for examining and optimizing processes in which several independent variables influence a measurable response. Its application enables the researcher to establish relationships between process variables and the response and to determine factor combinations capable of producing a desirable outcome. RSM is also applicable to product development, formulation, design improvement, and process enhancement (Montgomery, 2020).

Where experimental resources are inadequate for determining the absolute optimum response, RSM can still provide useful information about the behaviour of the system. If a known deterministic relationship exists between the response and experimental factors, suitable factor levels may be established mathematically. However, practical systems frequently exhibit complicated or poorly understood relationships. In such cases, empirical modelling provides a suitable means of representing the response and identifying favourable operating conditions (Khuri & Cornel, 1996).

Central Composite Design (CCD) is a commonly applied RSM technique for developing second-order mathematical models. It combines a two-level factorial arrangement with centre and axial points and is particularly useful for studying quadratic relationships. Box and Wilson introduced the design as an alternative to conventional three-level factorial arrangements (Box & Wilson, 1951). The principal components of a CCD are the factorial portion, centre points, and axial points. The factorial component may be a complete or fractional (2^k) arrangement, with factor levels normally represented by -1 and $+1$. Centre points are located at the middle of the experimental region, whereas axial points are positioned along the factor axes at predetermined distances from the centre. Together, these components provide information for estimating main, interaction, and quadratic effects (Montgomery, 2020; Khuri & Cornel, 1996).

Box–Behnken Design, developed by Box and Behnken (1960), is another experimental strategy used in Response Surface Methodology. Its construction combines two-level factorial arrangements with balanced incomplete block designs (BIBD), producing a systematic distribution of experimental points within the factor space. A major benefit of BBD is its efficiency in terms of experimental runs. It can provide adequate information for response-surface modelling without requiring as many experimental observations as some alternative designs. Consequently, it is useful where experimental materials, time, labour, or financial resources are limited (Montgomery, 2020; Myers et al., 2016).

Conventional RSM designs such as CCD and BBD may become unsuitable when an experiment involves complex restrictions. Although individual factors may have defined upper and lower limits, these limits may not adequately represent the actual feasible experimental region. Consequently, conventional designs can generate experimental points in areas that are impractical, unsafe, or impossible to investigate.

Computer-generated optimal designs address this limitation by using algorithms to select experimental points according to the objectives and restrictions of a particular investigation. Their application is especially valuable where standard RSM designs cannot accommodate complicated constraints (Anderson & Whitcomb, 2017; 2007).

Such designs are also applicable where the required predictive model is more complex than a conventional quadratic model. For example, prior process knowledge may indicate the need for cubic terms, for which CCD or BBD may not provide sufficient information. Optimal designs can additionally accommodate categorical factors, such as different catalyst types or configurations, alongside quantitative process variables (Anderson & Whitcomb, 2007).

Response Surface Methodology has been applied to concrete and cement-based materials for optimizing compositions and predicting engineering performance. Sridhar et al. (2022) used RSM

within a Design of Experiment framework to optimize the flexural moment of ferrocement composites containing galvanized square welded mesh and steel slag. A Central Composite Design was employed to determine suitable combinations of the investigated variables. Analysis of variance showed that the developed model was statistically significant, with a reported p-value below 0.05. The prediction-related errors were also reported to be below 20%, while the predicted responses corresponded satisfactorily with the experimental observations. The volume fraction of galvanized square welded mesh was identified as an important variable affecting flexural moment. Surface-response plots, Pareto analysis, and regression results further established its substantial influence on the flexural behaviour of the composites (Sridhar et al., 2022).

Kostić et al. (2016) applied RSM together with Monte Carlo simulation to develop a robust procedure for estimating concrete strength. Their model expressed concrete strength in relation to unit mass, water-to-cement ratio, age, and superplasticizer content. The resulting model demonstrated satisfactory accuracy during calibration and verification. Importantly, the verification stage used observations that were excluded from model development, thereby providing an independent evaluation of model performance. The residual analysis indicated normality during both stages, while uncertainty in the estimated coefficients had no statistically significant influence on concrete strength. The researchers also found that laboratory measurement variations had limited effects on predicted compressive strength, demonstrating the robustness of the developed approach.

Agarwal and John (2004) developed a reliability-based multidisciplinary design optimization framework using approximation techniques. Response Surface Approximations (RSA) were employed to represent limit-state functions and estimate failure probabilities while reducing the computational requirements associated with conventional Reliability-Based Design Optimization (RBDO). The methodology was tested using two problems. For a small analytical problem involving two design variables, significant computational savings were not obtained because constructing the RSA could require more effort than directly evaluating the reliability constraints. In contrast, substantial computational savings were achieved for a higher-dimensional multidisciplinary HPLC structural problem involving configuration, structural, and dynamic disciplines. The findings demonstrate that response-surface approximations can be particularly advantageous for computationally demanding multidisciplinary optimization problems.

Bamboo is a rapidly renewable lignocellulosic material belonging to the grass family Poaceae. Its combination of relatively low density, rapid growth, high specific strength and widespread availability has encouraged its investigation as an alternative construction material. However, bamboo should not be treated as a uniform engineering material because its physical and mechanical characteristics vary substantially with species, age, moisture content, growing conditions, culm geometry and position along the culm (Trujillo, 2013; Sharma et al., 2015).

Several species have been investigated for structural applications, including *Bambusa vulgaris*, *Bambusa balcooa*, *Bambusa bambos*, *Dendrocalamus asper*, *Dendrocalamus strictus*, *Guadua angustifolia* and *Phyllostachys edulis*. *Guadua angustifolia* has received considerable attention in structural engineering because of its favourable strength-to-weight characteristics, while *Dendrocalamus* and *Bambusa* species are important in Asian and African environments. *Bambusa vulgaris* is of particular interest in tropical regions because of its widespread availability, including in Nigeria. Nevertheless, the use of any local species in structural applications requires experimental characterization rather than assuming properties reported for another species. Reviews have identified species, age, location and treatment as important sources of variation in bamboo properties (Sharma et al., 2015; Maria et al., 2023).

The longitudinal arrangement of bamboo fibres gives the material comparatively high tensile strength parallel to the culm axis. This characteristic makes bamboo attractive for tensile applications. Conversely, its transverse properties are considerably weaker, and the natural hollow geometry and variation in wall thickness create additional challenges for structural detailing and connections.

The untreated bamboo culm is vulnerable to biological deterioration, particularly fungal attack and insects such as termites and beetles. Its relatively high concentration of starch and other soluble constituents contributes to this vulnerability. Consequently, treatment is generally considered necessary where bamboo is intended for long-term structural service.

Bamboo preservation techniques can broadly be divided into traditional/natural, chemical and physical treatments. Traditional methods include water immersion, smoking and controlled drying,

while chemical preservation commonly involves boron-based preservatives, particularly borax-boric acid formulations. Other approaches include pressure impregnation and copper-based preservatives. Treatment effectiveness depends on the ability of the preservative to penetrate the bamboo wall; therefore, treatment of only the external surface may not provide adequate protection for structural applications. Recent reviews also emphasize the need to standardize treatment procedures because different treatment methods can produce different effects on strength and durability (Maria et al., 2023).

Adequate durability therefore requires more than chemical preservation. Bamboo should be properly seasoned, treated throughout its thickness and protected from persistent moisture. Appropriate construction detailing—including drainage, ventilation, protection from direct wetting and separation from the ground—is equally important. These considerations have become part of modern structural-bamboo guidance. ISO 22156:2021 specifically includes durability as one of the performance requirements for structural bamboo (International Organization for Standardization (ISO) 2021).

The durability problem becomes more complex when bamboo is used as reinforcement in concrete. The concrete environment is highly alkaline, while bamboo is a lignocellulosic material. Research has reported deterioration of bamboo under prolonged alkaline exposure, while moisture movement between bamboo and concrete can cause swelling, shrinkage and loss of bond. Archila et al. (2018) consequently identified durability, bond and dimensional stability as major obstacles to the reliable use of bamboo as conventional reinforcement.

The development of standards has been important in moving bamboo from traditional construction towards engineered applications. ISO 22156:2021, Bamboo structures—Bamboo culms—Structural design, provides design provisions for structures whose primary load-bearing members are round bamboo culms or certain shear-panel systems. Its scope covers mechanical resistance, serviceability and durability and is principally directed towards relatively small bamboo structures, including one- and two-storey buildings not exceeding 7 m in height (International Organization for Standardization (ISO) 2021).

Statistical modelling and optimization techniques, including response surface methodology (RSM), provide useful means of examining relationships among influencing factors, responses, constraints, and optimization objectives (Usman et al., 2021). In the context of bamboo-reinforced concrete, the absence of established design codes, together with the increasing emphasis on sustainable and environmentally responsible construction, creates a need for systematic approaches to material and structural development. RSM offers a promising approach for evaluating and optimizing the combined effects of several parameters governing the engineering characteristics and performance of bamboo-reinforced concrete beams. Such modelling can improve understanding of the contribution of individual variables and their interactions while reducing the number of experimental trials required within the experimental programme to develop a numerical representation of the governing mechanisms and procedures (Usman et al., 2021). Accordingly, the adoption of RSM is justified in the present study.

2. MATERIALS AND METHODS

2.1 Materials

The data used for the predictive modelling as well as the model validation were obtained from experimental programs reported in the previous sections of this research for flexural properties (deflection and failure load) at:

- (i) Curing age of 7-, 14-, 21- and 28-day;
- (ii) Concrete grades of 25 MPa;
- (iii) area of bamboo, 145-, 290-, 435-, 580mm² and
- (iv) area of steel 50.3-, 100.6-, 150.9- and 201.2mm².

The modelling and validation were performed using Design Expert Software developed by Stat-Ease.

2.2 Methodology

Model Equation

The true functional relationship is given in Equation 1 where ε represents the noise or error observed in the response y while the variables are $x_1, x_2, \dots, x_k$.

$$y = f(x, \theta) + \varepsilon \quad 1$$

The model is just a quantitative relationship (equation) between the response and the important design factors. In many cases, a low-order polynomial model will be appropriate. A first-order model in two variables is as given in Equation 2

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \dots + \beta_k x_k + \varepsilon \quad 2$$

where y is the response, the x 's are the design factors, the β 's are unknown parameters that will be estimated from the data in the experiment, and ε is a random error term that accounts for the experimental error in the system that is being studied. A common extension of the first-order model is to add an interaction term, as shown in Equation 3

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_{12} x_1 x_2 + \varepsilon \quad 3$$

where the cross-product term $x_1 x_2$ represents the two-factor interaction between the design factors. Because interactions between factors is relatively common, the first-order model with interaction is widely used. Higher-order interactions can also be included in experiments with more than two factors if necessary. A widely used model is the second-order model given in Equation 4.

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_{12} x_1 x_2 + \beta_{11} x_1^2 + \beta_{22} x_2^2 + \varepsilon \quad 4$$

When there is curvature in the system, then a polynomial of higher degree must be used, such as the third-order model

A third order equation usually means a cubic response-surface model. It is used when the relationship between experimental factors and the measured response is sufficiently nonlinear that a first- or second-order model cannot adequately describe it as shown in Equation 5.

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_{12} x_1 x_2 + \beta_{11} x_1^2 + \beta_{22} x_2^2 + \beta_{112} x_1^2 x_2 + \beta_{122} x_1 x_2^2 + \beta_{111} x_1^3 + \beta_{222} x_2^3 + \varepsilon \quad 5$$

Optimal (Custom) Design

In RSM, optimal (Custom) Design is used to determine the relationship between responses and independent variables (factors). Optimal (Custom) Design (OCD) is a flexible design structure to accommodate custom models, categoric factors, and irregular (constrained) regions. Custom designs are used when the process requires adjustments to the experimental data that cannot be accommodated by a standard design.

Fitting Response Curves and Surfaces

Many experiments involve at least one quantitative factor. It can be useful to fit a response curve to the levels of a quantitative factor so that the experimenter has an equation that relates the response to the factor. This equation might be used for interpolation, that is, for predicting the response at factor levels between those actually used in the experiment. When at least two factors are quantitative, a response surface can be fitted for predicting (y) at various combinations of the design factors. The dependent variables (responses) are deflection and flexural load, while the independent variables (factors) are: curing age and area of reinforcement.

Model validation

The models developed from the experimental data were validated via Optimal Custom Design in RSM of Design-Expert. The validation was done using the flexural properties (deflection and failure load) of reinforced bamboo and steel reinforced concrete beams at targeted concrete grades of 25 MPa mix design.

3 RESULTS AND DISCUSSION

3.1 Experimental Results and discussions

The experimental part of this research includes the analysis of results of flexural properties of bamboo and steel reinforced concrete specimens. The experimental data obtained from the flexural

behaviour of bamboo and steel reinforced concrete beams under flexure at grades 25 MPa concrete were used as input data for the comparisons in RSM for developing the model.

3.2 Model development

The controllable variable and the design summary of optimal custom design (OCD) used in this experiment is as shown in table 1. RSM allowed estimation of the linear, quadratic, cubic and factor interaction effects of the variables on the flexural properties. To obtain the optimum response, the suggested cubic model (third order polynomial) given in Equation 5 was used:

Table 1: Controllable variable in optimal custom design

No	Factor	Name	Units	Variation interval			
1	A	Age	day	7	14	21	28
2	B	Area of Bamboo	mm^2	145	290	435	580
3	S	Area of Steel	mm^2	50.3	100.6	160.9	201.2

3.3 Sequential Model Sum of Squares (SMSS)

Table 2 and 3 show the full details of the SMSS dataset for response 1 (deflection) and response 2 (failure load) at grade 25 MPa concrete with recommended regression model. It can be seen that third-order equations were suggested as the highest order of polynomial that was not aliased. Thus, the models are most suitably described with cubic polynomial model.

Table 2: SMSS for deflection at grade 25 MPa concrete mix design

Source	Bamboo reinforced beam					Steel reinforced beam					
	Sum of Squares	df	Mean Square	F-value	p-value	Sum of Squares	df	Mean Square	F-value	p-value	
Mean	874.27	1	874.2			1434.9	1	1434.9			
Linear	331.89	2	165.9	140.1	<0.0001	544.73	2	272.37	140.1	<0.0001	
2FI	31.41	1	31.41	297.8	<0.0001	51.55	1	51.55	297.8	<0.0001	
Quadratic	0.4643	2	0.232	2.43	0.1082	0.7621	2	0.3811	2.43	0.1082	
Cubic	2.12	4	0.529	31.34	< 0.0001	3.48	4	0.8688	31.34	<0.0001	Suggested
Quartic	0.3707	3	0.123	2584.1	< 0.0001	0.6085	3	0.2028	2584	<0.0001	Aliased
Residual	0.0009	19	0.000			0.0015	19	0.0001			
Total	1240.5	32	38.77			2036.0	32	63.63			

Table 3: SMSS for failure load at grade 25 MPa concrete mix design

Source	Bamboo reinforced beam					Steel reinforced beam					
	Sum of Squares	df	Mean Square	F-value	p-value	Sum of Squares	df	Mean Square	F-value	p-value	
Mean	1421.11	1	1421.11			3201.27	1	3201.27			
Linear	353.39	2	176.69	350.40	< 0.0001	639.79	2	319.90	315.15	< 0.0001	
2FI	7.25	1	7.25	27.49	< 0.0001	12.68	1	12.68	21.18	< 0.0001	
Quadratic	5.02	2	2.51	27.59	< 0.0001	11.74	2	5.87	30.41	< 0.0001	
Cubic	2.14	4	0.5342	51.83	< 0.0001	4.58	4	1.14	56.89	< 0.0001	Suggested
Quartic	0.2267	3	0.0756	4.737E+05	< 0.0001	0.4425	3	0.1475	4.930E+05	< 0.0001	Aliased
Residual	3.032E-06	19	1.596E-07			5.685E-06	19	2.992E-07			
Total	1789.12	32	55.91			3870.50	32	120.95			

3.4 Analysis of variance (ANOVA)

The analysis of variance is a formal way to determine which factor effects are nonzero. It is probably the most useful technique in the field of statistical inference. Table 4 and 5 show the full ANOVA results for Cubic models suggested in table 2 and 3. It was shown that the models' terms for bamboo and steel reinforced concrete beams for the two responses (deflection and failure load) achieve significance at the 0.05 p-value threshold (95% confidence interval). This outcome translates to 99.99% confidence (calculated by taking $[1 - 0.0001] \times 100$) that the cubic model coefficient is not zero. In table 4 the models' terms: A, B/S, A(B/S), A², B²/ S², A³, B³/ S³ have significant effects on response (deflection) but A² is marginally significant. The models' terms: A, B/ S, A(B/S), A², A²(B/S) and A³ also have significant effect on response (failure load) as shown in table 5.

Table 4: ANOVA of the cubic model for response 1 (deflection) at grade 25 MPa

Source	Bamboo reinforced beam					Steel reinforced beam					
	Sum of Squares	df	Mean Square	F-value	p-value	Sum of Squares	df	Mean Square	F-value	p-value	
Model	365.79	7	52.26	2742.03	<0.0001	600.38	7	85.77	2742.03	<0.0001	significant
A-Age	9.06	1	9.06	475.23	<0.0001	14.86	1	14.86	475.23	<0.0001	
B/S- Area	24.84	1	24.84	1303.17	<0.0001	40.76	1	40.76	1303.17	<0.0001	
A(B/S)	31.41	1	31.41	1648.02	<0.0001	51.55	1	51.55	1648.02	<0.0001	
A ²	0.1146	1	0.1146	6.01	0.0218	0.1881	1	0.1881	6.01	0.0218	
B ² / S ²	0.3497	1	0.3497	18.35	<0.0003	0.5740	1	0.5740	18.35	<0.0003	
A ³	0.2831	1	0.2831	14.85	<0.0008	0.4646	1	0.4646	14.85	<0.0008	
B ³ / S ³	1.75	1	1.75	91.75	<0.0001	2.87	1	2.87	91.75	<0.0001	

Table 5: ANOVA of the cubic model for response 2 (failure load) at grade 25 MPa

Source	Bamboo reinforced beam					Steel reinforced beam					
	Sum of Squares	df	Mean Square	F-value	p-value	Sum of Squares	df	Mean Square	F-value	p-value	
Model	367.78	6	61.30	6688.15	<0.0001	668.78	6	111.46	6239.18	<0.0001	significant
A-Age	0.3957	1	0.3957	43.17	<0.0001	0.7371	1	0.7371	41.26	<0.0001	
B/S-Area	144.72	1	144.72	15789.9	<0.0001	256.66	1	256.66	14366.5	<0.0001	
A(B/S)	7.25	1	7.25	790.54	<0.0001	12.68	1	12.68	709.61	<0.0001	
A ² / S ²	5.01	1	5.01	546.99	<0.0001	11.74	1	11.74	656.98	<0.0001	
A ² (B/S)	1.13	1	1.13	123.69	<0.0001	2.08	1	2.08	116.65	<0.0001	
A ³	1.00	1	1.00	109.40	<0.0001	2.49	1	2.49	139.48	<0.0001	

3.5 Selecting a good predictive model

The regression diagnostics, shown in Table 4 and 5 list the statistical parameters calculated at the 95% confidence level, commonly used in analyses of statistical data. The regression model in terms of both actual and coded variables from the above analysis for bamboo and steel reinforced concrete beam at 25 MPa concrete grade, are as given in Equation 6 to 13 below:

Where D = deflection; F.L. = failure load; A, B and represent the design factors: age and area of bamboo and area of steel, respectively. This regression model can be used to predict the flexural properties of bamboo and steel reinforced beams with the two factors (age and area of reinforcement).

1 CODED MATHEMATICAL MODEL FOR BAMBOO AND STEEL CONCRETE BEAMS RESPECTIVELY

25MPa concrete grade

$$\text{Def. (Bam)} = 5.43 + 2.54A + 4.21B + 1.78AB - 0.13A^2 - 0.24B^2 + 0.47A^3 - 1.18B^3 \quad 6$$

$$\text{Def. (Ste)} = 6.96 + 3.26A + 5.39S + 2.28AS - 1.17A^2 - 0.30S^2 + 0.61A^3 - 1.51S^3 \quad 7$$

2 ACTUAL MATHEMATICAL MODELS FOR BAMBOO AND STEEL CONCRETE BEAMS RESPECTIVELY

25MPa concrete grade

$$\text{Def. (Bam)} = 1.36 + 0.38A - 3.58(10^{-2})B + 7.81(10^{-4})AB - 2.27(10^{-2})A^2 + 1.19(10^{-4})B^2 + 4.09(10^{-4})A^3 - 1.14(10^{-7})B^3 \quad 8$$

$$\text{Def. (Ste)} = 1.75 + 0.48A - 0.13S + 2.88(10^{-3})AS - 2.91(10^{-2})A^2 + 1.27(10^{-3})S^2 + 5.24(10^{-4})A^3 - 3.51(10^{-6})S^3 \quad 9$$

3 CODED MATHEMATICAL MODEL FOR BAMBOO AND STEEL CONCRETE BEAMS RESPECTIVELY

25MPa concrete grade

$$\text{Fail. Load (Bam)} = 7.16 + 0.53A + 4.57B + 0.86AB - 0.89A^2 - 0.57A^2B + 0.89A^3 \quad 10$$

$$\text{Fail. Load (Ste)} = 10.76 + 0.72A + 6.08S + 1.13AS - 1.36A^2 - 0.77A^2S + 1.4A^3 \quad 11$$

4 ACTUAL MATHEMATICAL MODELS FOR BAMBOO AND STEEL CONCRETE BEAMS RESPECTIVELY

25MPa concrete grade

$$\text{Fail. Load (Bam)} = -2.93 + 0.60A + 7.18(10^{-3})B + 1.20(10^{-3})AB - 3.99(10^{-2})A^2 - 2.40(10^{-5})A^2B + 7.69(10^{-4})A^3 \quad 12$$

$$\text{Fail. Load (Ste)} = -4.16 + 1.03A + 2.72S + 4.67(10^{-3})AS - 6.44(10^{-2})A^2 - 9.30(10^{-5})A^2S + 1.21(10^{-3})A^3 \quad 13$$

3.6 Diagnosing residuals to validate statistical assumptions

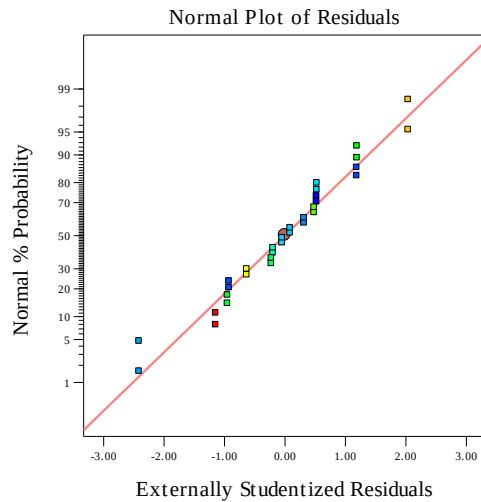
Tables 6 gives the fit summary of the models in terms of R^2 , Adjusted R^2 , predicted R^2 and Adequate precision. The adjusted R^2 values for all the models are between 0.9984 - 0.9992 which is an indication of good predictive powers of the models. The indicator of the quality of the regression models shows that Predicted R^2 values are in reasonable agreement with the Adjusted R^2 ; that is., the difference in values is less than 0.2 for the models, which is an indication of a good predictive power of the models. Adequate Precision which is a measure of the signal to noise ratio is also a confirmation tool. A ratio greater than 4 confirms adequate signals and is also an indication of good predictive powers of the models. These suggest that the predicted values obtained based on the regression models are very much close to the values obtained from the experiment.

Table 6: Fit summary of the models

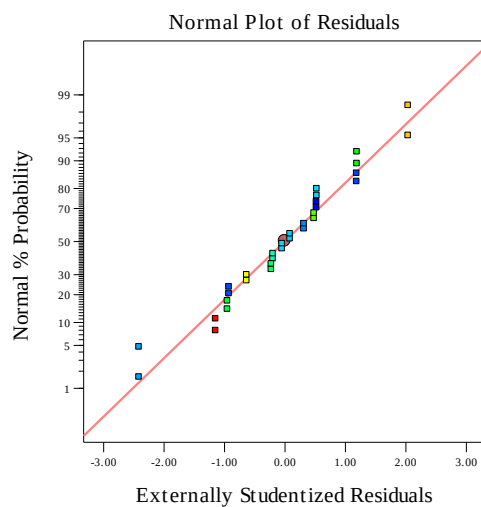
Beam	Flexural Property	Concrete Strength	R^2	Adjusted R^2	Predicted R^2	Adj(R^2) - Pred.(R^2) < 0.2	Adeq Precision > 4
Bamboo	Deflection	25	0.9988	0.9984	0.9978	$6(10^{-4})$	175.18
	Failure Load	25	0.9994	0.9992	0.9989	$9(10^{-4})$	242.13
Steel	Deflection	25	0.9988	0.9984	0.9978	$6(10^{-4})$	175.18
	Failure Load	25	0.9993	0.9992	0.9988	$4(10^{-4})$	238.05

For statistical purposes, it's expected that residuals are normally distributed and independent with constant variance. Two plots are suggested for checking the statistical assumptions: (i) Normal plot of residuals, (ii) Predicted versus Actual level.

Normal plot of residuals: These are shown in Figure 1 to 2. The over-all idea from this display is that the error distribution is approximately normal. The tendency of the normal probability plot to bend down slightly on the left side and upward slightly on the right side implies that the tails of the error distribution are somewhat thinner than would be anticipated in a normal distribution; that is, the largest residuals are not quite as large (in absolute value) as expected. These plots are used to predict the adequacy of the models adopted. In this case, giving the plots a pencil test, the deviations from linear (the normal line) are very minor, so the assumption of normality is supported. The normal plot of residual for bamboo reinforced concrete looks very much alike to that of steel reinforced concrete beam. This show that the flexural properties of bamboo reinforced concrete beam is almost the same as that of steel reinforced concrete beam.

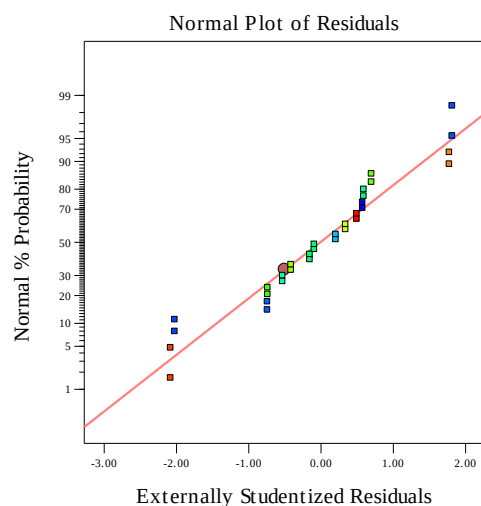


(a) Bamboo reinforced concrete beam

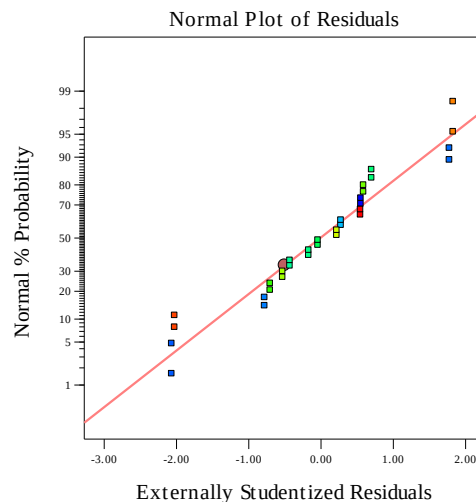


(b) Steel reinforced concrete beam

Figure 1 (a & b): Normal plot of residual for response 1 (deflection) at grade 25 MPa concrete



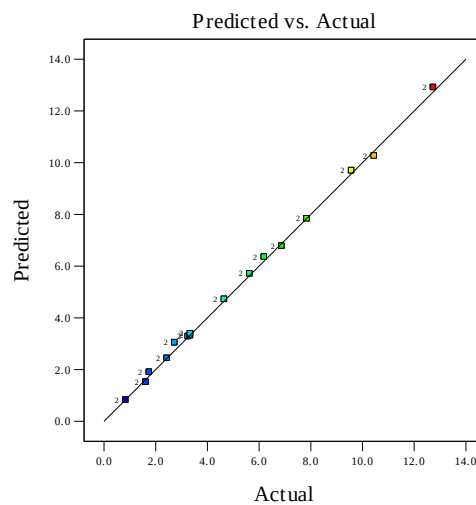
(a) Bamboo reinforced concrete beam



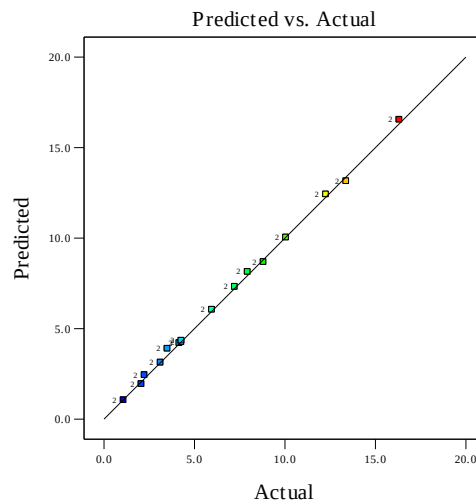
(b) Steel reinforced concrete beam

Figure 2 (a & b): Normal plot of residual for response 2 (failure load) at grade 25 MPa concrete

Predicted versus Actual level: The fit of predicted versus actual data for the cubic models in this study look very good as shown in Figure 3 to 4. In this case, the models are clearly confirmed as these plots line up nicely as expected from a normal distribution. The pencil test for linearity holds good for all the models as there are no deviations from the normal line. This further confirms that the assumption of normality is supported and the adequacy of the cubic models for predicting the flexural properties of bamboo and steel reinforced concrete beam. In turn, this suggests that bamboo is a substitute to steel reinforcement.

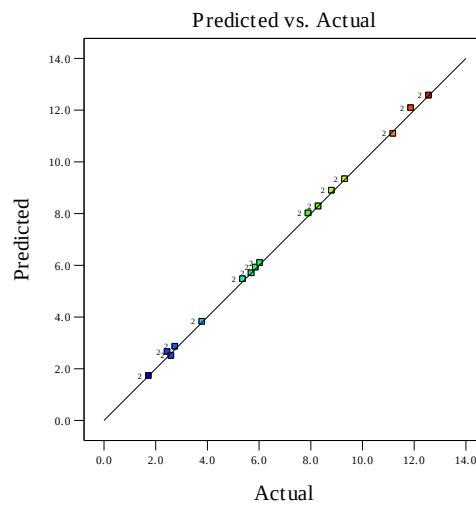


(a) Bamboo reinforced concrete beam

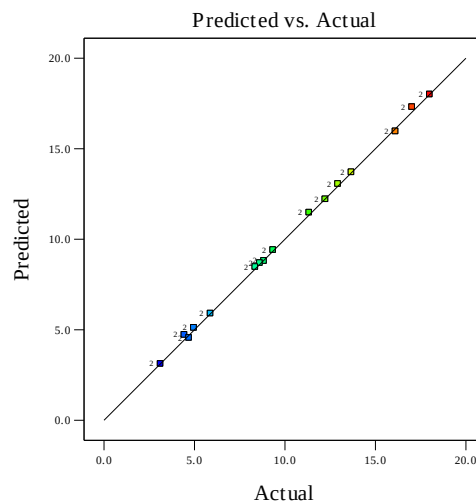


(b) Steel reinforced concrete beam

Figure 3 (a & b): Predicted vs Actual plot for response 1 (deflection) at Grade 25 MPa concrete



(a) Bamboo reinforced concrete beam



(b) Steel reinforced concrete beam

Figure 4 (a & b): Predicted vs Actual plot for response 2 (failure load) at Grade 25 MPa concrete

3.7 Surface plots of flexural properties of bamboo and steel reinforced concrete beams

The 3D surface plots show how the predicted responses (deflection and failure load) vary for the two factors (curing age and area of reinforcement) as shown in Figures 5 and 6. Results from the Figures show that the deflection and failure load of both bamboo and steel reinforced concrete beams increase with: increase in curing age, area of reinforcements (bamboo/steel) at 25Mpa concrete grade. However, it was observed from these figures that the difference in the values of bamboo reinforced concrete beams and steel reinforced concrete beams were caused by the ultimate strength of the steel reinforcement. Thus, the values of deflection and failure load in bamboo reinforced concrete beam are very close with that of the steel reinforced concrete beam.

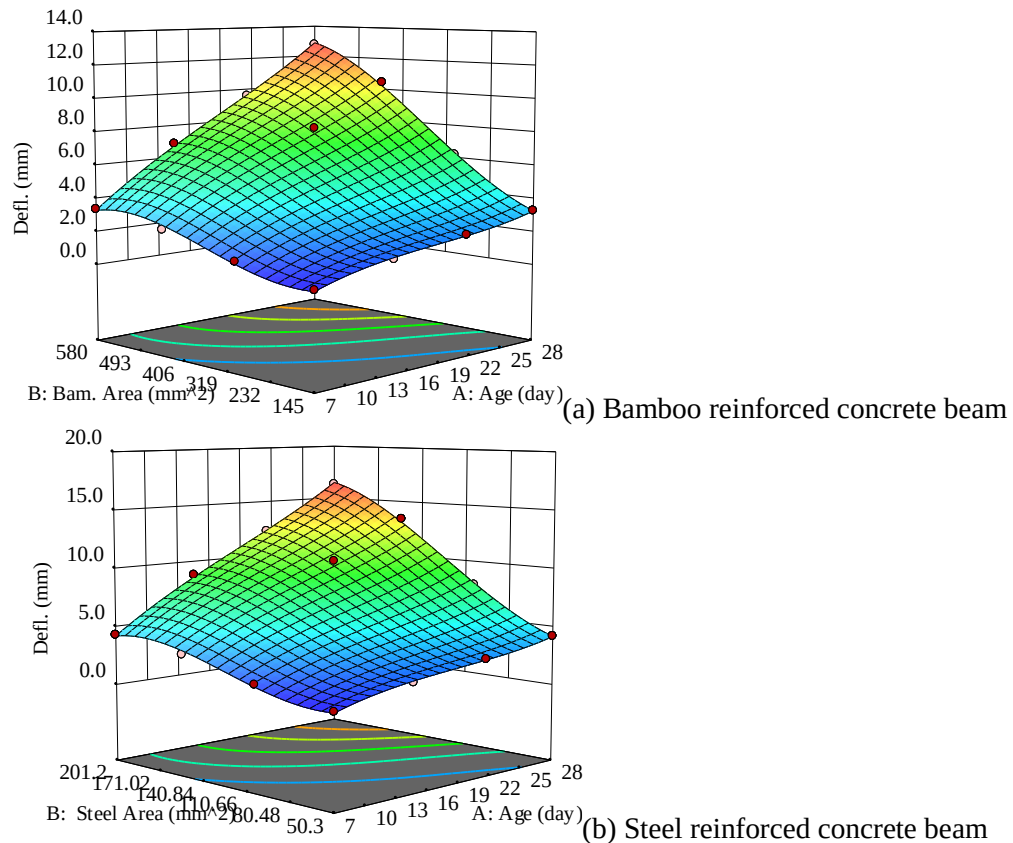
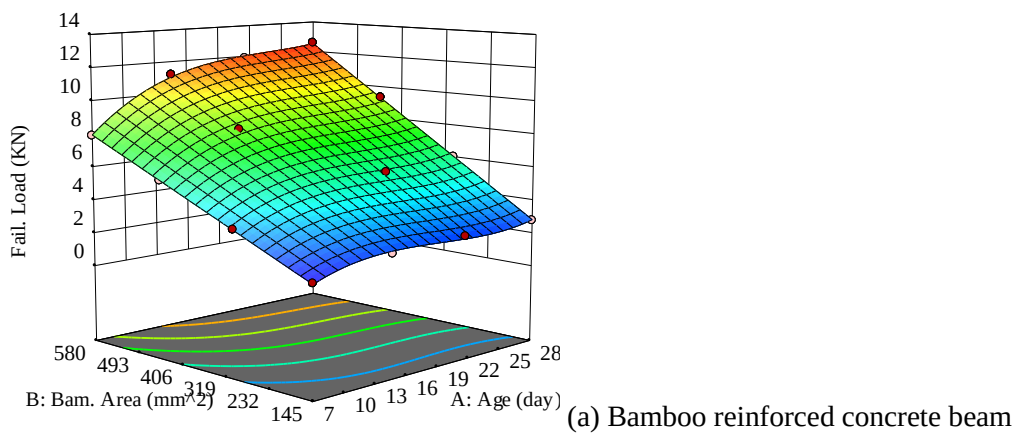
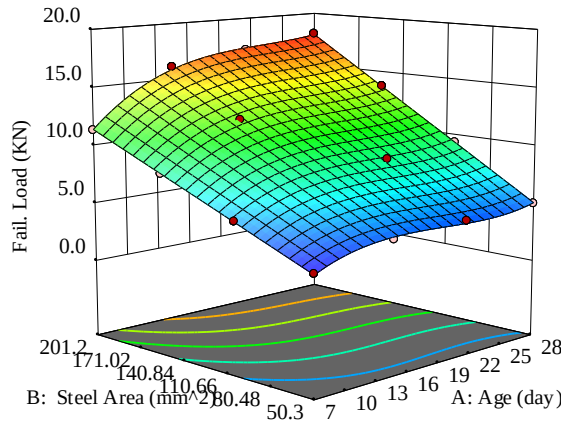


Figure 5 (a & b): 3D Surface plot for response 1 (deflection) at grade 25 MPa concrete





(b) Steel reinforced concrete beam

Figure 6 (a & b): 3D Surface plot for response 2 (failure load) at grade 25 MPa concrete

4.8 Evaluation of multiple responses via desirability function

Optimization of bamboo and steel reinforced concrete beams were done such that for response: deflection was minimized while failure load was maximized; for factors: area of bamboo and steel were maximized while age was set at default that is, in range of actual values in order to obtain the optimal curing age and area of bamboo/steel gave the best values. The optimization goals and constraints are as shown in table 7

Table 7: Optimization goals and constraints for grade 25 MPa concrete

Grade of Concrete	Bar	Name	Goal	Lower Limit	Upper Limit	Lower Weight	Upper Weight	Importance
grade 25 MPa	Bamboo	A:Age	is in range	7	28	1	1	4
		B:Bam. Area	maximize	145	580	1	1	4
		Deflection	minimize	0.8611	12.763	1	1	4
		Fail. Load	maximize	1.75	112.58	1	1	4
	Steel	A:Age	is in range	7	28	1	1	4
		B: Steel Area	maximize	50.3	201.2	1	1	4
		Deflection	minimize	1.10329	16.350	1	1	4
		Fail. Load	maximize	3.15	18.312	1	1	4

The result from table 7 shows: the importance placed on all inputs combinations; the curing age; area of bamboo and steel were set at 7-28 days; 145-580mm² and 50.3-201.2mm² respectively in which the best optimized flexural properties values were selected.

The summary of the optimized result is present in table 8. From the optimal parameters, approximately 10 days curing age was achieved for both bamboo and steel reinforced concrete showing that the method of curing was the same for both specimens. The deflection parameters show that steel reinforced concrete beam is 24.67 percent more ductile than bamboo concrete beam, while the failure load parameters show that steel reinforce concrete beam can sustain load up to 31.70 percent more than bamboo reinforced concrete beams. The large areas of bamboo is to complement the yield strength of bamboo due to its brittle nature. The desirability values are extremely close (0.785 vs. 0.782), showing how well the combinations of input factors satisfied the responses simultaneously. It means that the proposed factor combination provides a strong overall compromise among the response objectives. From the foregoing, Bamboo is the next substitute to steel for low-cost housing units.

Table 8: Summary of optimized result of bamboo and steel reinforced concrete beams

Grade of Concrete	Bar	Number of solutions	Age (day)	Area (Bamboo) (mm ²)	Deflection (mm)	Flexural Load (KN)	Desirability	
25 MPa	Bamboo	6	9.588	580.000	4.692	9.471	0.785	Selected
	Steel	11	9.926	201.200	6.229	13.865	0.782	Selected

CONCLUSION AND RECOMMENDATION

5.1 Conclusion

- (i). The results of the mathematical models show that the flexural properties (deflection and failure load) of bamboo and steel reinforced concrete beams can be predicted by third-order polynomial equation (cubic models).
- (ii). The P-values of all the models achieved significance at $0.0001 < 0.05$ (95% threshold confidence interval). This outcome translates to 99.99% confidence (calculated by taking $[1 - 0.0001]$ times 100). Thus, the cubic models are statistically significant.
- (iii). The adjusted R^2 values are in reasonable agreement with the predicted R^2 for flexural properties as their difference ($Adjusted R^2 - Predicted R^2 \leq 0.2$). Also, the adequate precision values are greater than four, which is an indicator that the predicted values obtained based on the regression models are very much close to that obtained from the experiment.
- (iv). The deflection parameters show that steel reinforced concrete beam is 24.67 percent more ductile than bamboo concrete beam, while the failure load parameters show that steel reinforced concrete beam can sustain load up to 31.70 percent more than bamboo reinforced concrete beams.
- (v). The desirability values showed that the combinations of input factors satisfied the responses simultaneously i.e., the proposed factor combination provides a strong overall compromise among the response objectives.

5.2 Recommendation

1. Validation programme should be extended to different concrete grades, bamboo species, bamboo ages, treatment methods and reinforcement configurations so as to establish the limits of applicability and reliability of the predictive models.
2. The developed RSM equations should subsequently be incorporated into a numerical/design software framework that can assist engineers in predicting structural performance and selecting appropriate bamboo reinforcement quantities.
3. Larger-scale experimental studies involving beams, slabs, columns and complete structural systems should be conducted to establish whether the laboratory-scale relationships remain valid at structural scale.
4. Field trials should monitor cracking, deflection, moisture effects, bond behaviour, durability, deterioration and long-term load response over an extended period. The field programme should also evaluate constructability, availability of suitable bamboo, treatment requirements, labour requirements, cost, maintenance and overall economic viability.

REFERENCES

- Agarwal, H. & John, E. R. (2004). Reliability based design optimization using response surfaces in application to multidisciplinary systems. *Engineering Optimization*, 36 (4), 291--311.
- Anderson, J. M. & Whitcomb, J. P. (2007). *DOE simplified - practical tools for effective experimentation (PDFDrive)*. CRC Press.
- Anderson, J. M. & Whitcomb, J. P. (2017). *RSM Simplified: Optimizing Processes Using Response Surface Methods for Design of Experiments*. Productivity Press.
- Archila, H., Kaminski, S., Trujillo, D., Zea Escamilla, E., & Harries, K. A. (2018). Bamboo reinforced concrete: A critical review. *Materials and Structures*, 51 (102) <https://doi.org/https://doi.org/10.1617/s11527-018-1228-6>
- Box, G. EP. & Behnken, D. W. (1960). Some new three level designs for the study of quantitative variables. *Technometrics*, 2 (4), 455--475.
- Box, G. EP. & Wilson, K. B. (1951). On the experimental designs for exploring response surfaces. *Ann Math Stat*, 13, 1--45.
- Burgoyne, C. & Balafas, I. (2007). 8th International Symposium on Fiber Reinforced Polymer Reinforcement for Concrete Structures. 16-18 July 2007 , Patras.
- Chow, H. K. 1914. "Bamboo As A Material for Reinforcing Concrete."
- Ghavami, K. (1995). Ultimate load behaviour of bamboo-reinforced lightweight concrete beams. *Cement and concrete composites*, 17 (4), 281--288.
- International Organization for Standardization (ISO). 2021. "ISO 22156: 2021Bamboo structures—Bamboo culms—Structural design." Geneva: ISO.
- Khuri, I. A. & Cornell, A. J. (1996). *Response Surfaces Designs and Analyses*. CRC Press.
- Kostić, S., Vasovi, N., & Marinković, B. (2016). Robust optimization of concrete strength estimation using response surface methodology and Monte Carlo simulation. *Engineering Optimzation*, 49 (5), 864--877.
- Maria, F. VA., Maria, E. PS., Daniel, N. RV., & Jason, O. MC. (2023). Bamboo as Sustainable Building Materials: A Systematic Review of Properties, Treatment Methods, and Standards. *Buildings*, 13 (10), 2449. <https://doi.org/https://doi.org/10.3390/buildings13102449>
- McCormac, J. C. & Brown, R. H. (2014). *Design of Reinforced Concrete*. 9th edition. USA: Wiley & Sons.
- Montgomery, D. C. (2020). *Design and analysis of experiments*. Wiley.
- Myers, R. H., Montgomery, D. C., & Anderson-Cook, C. M. (2016). *Response surface methodology: process and product optimization using designed experiments*. John Wiley & Sons.
- Pacheco-Torgal, F. & Labrincha, J. A. (2013). The future of construction materials research and the seventh UN Millennium Development Goal: A few insights. *Construction and Building Materials*. 40, pp. 729--737, 40, 729--737.
- Sharma, A. (1990). Bamboo-reinforced concrete beams In: Vandepitte, D. *Mixed Structures, including New Materials*. Brussels, 1990. Zurich: International Association for Bridge and Structural Engineering, 677--682.
- Sharma, B., Gattoo, A., Bock, M., & Ramage, M. (2015). Engineered bamboo for structural applications. *Construction and Building Materials* (81), 66--73.
- Sridhar, J., Shinde, G. B., Vivek, D., Naseem, K., Gaur, P., Patil, P. P., & Tesema, M. T. (2022). Response surface methodology approach to predict the flexural moment of ferrocement composites with weld mesh and steel slag as partial replacement for fine aggregate. *Advances in Materials Science and Engineering*, 2022, 1--9.
- Trujillo, D. (2013). *Bamboo Structures*. In R. D. K. Misra (Ed.), *Modern Bamboo Structures*. CRC Press.

USGS (2019). Mineral Commodity Summaries 2019. *Reston: U.S. Geological Survey*

Usman, A., Sutanto, M. H., Napish, M. B., & Yaro, N. SA. (2021). Response surface methodology optimization in asphalt mixtures: A review. *Response Surface Methodology in Engineering Science*.