
PROBABILISTIC RELIABILITY ANALYSIS OF REINFORCED METAKAOLIN-BASED GEOPOLYMER CONCRETE BEAMS SUBJECTED TO VARIATIONS IN GEOPOLYMER MIX PARAMETERS

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ABSTRACT

The study investigated the probabilistic effect of NaOH Molarity (M_c), $Na_2SiO_3:NaOH$ (AL_r), and alkaline-liquid:Metakaolin (AL_m), on the reliability of metakaolin-based geopolymer concrete beams reinforced with two (2) 8 mm bars as tension reinforcement under a failure load of 39 kN. The assessment was performed using the First Order Reliability Method (FORM). Compressive strength models were developed for the investigated variables, which were incorporated into the failure modes limit-state equations through the FORM5 package to evaluate the beam's reliability. The probabilistic analyses were conducted by varying M_c (6-16 M), AL_r (0.5-2.5) and AL_m (0.4-1.0). Results from the analyses showed that the models explained 63.4% and 93.2% of the compressive strength variations, respectively, for M_c/AL_r and AL_m/W_c . Additionally, increasing M_c from 6 M to 10 M improved the reliability of the Reinforced Metakaolin Based Geopolymer concrete (RMBGC), whereas further increase to 12 M and beyond reduced reliability. Increasing AL_r from 0.5 to 2.5 and AL_m from 0.4 to 1.0 consistently enhanced the reliability. Among all variables, AL_m exerted the greatest influence, producing the largest improvement in safety index across all failure modes. It was also noted that the Moment failure mode was most sensitive to mix-parameter variations, followed by shear, while deflection was least affected. In general, the study demonstrates that optimising AL_m , AL_r , and M_c (approximately 10 M) provides the greatest improvement in the reliability and structural safety of metakaolin-based geopolymer reinforced concrete beams.

Keywords: Reinforced Metakaolin-Based Geopolymer concrete; First-Order Reliability Method (FORM); Safety index; Mix design parameters

1.0 INTRODUCTION

Research into alternatives to ordinary Portland cement (OPC) has been ongoing due to the fact that OPC is not sustainable. This is connected to the amount of carbon dioxide emitted during cement production. One of the most promising alternatives is geopolymer cement, which is produced by activating aluminosilicate materials such as metakaolin with alkaline solutions. Unlike OPC concrete, the binding phase in geopolymer concrete is formed through a geopolymerization process. The resulting concrete from the geopolymerisation has better mechanical properties, improved durability, and lower environmental impact (Davidovits, 1991).

The performance of geopolymer concrete depends largely on its mix composition. It was reported that the chemical composition in terms of the concentration of sodium hydroxide, the ratio of sodium silicate to sodium hydroxide, the alkaline liquid-to-binder ratio, and the water content significantly influences the geopolymerization reaction and consequently the strength and durability of the hardened product (Diaz et al., 2010). For metakaolin-based geopolymer concrete in particular, these parameters determine the extent of aluminosilicate dissolution, gel formation, and the development of compressive strength. Hence, a proper selection of these mix variables is important in achieving the desired structural performance.

Several studies have investigated the influence of mix proportions on the compressive strength and durability of geopolymer concrete. The studies did not incorporate uncertainties and did not study the effects on the safety of a structural member. This is because, in reality, material properties are associated with uncertainties that may influence the behavior and safety of structural members. These uncertainties can only be adequately considered through probabilistic methods, where the variability of the governing parameters is incorporated into the structural assessment rather than relying solely on characteristic values. A few studies checked the effect of the variability of the concrete mix materials on the reliability of structural members; some of these studies include Adamu et al, (2023), who considered the effect of mix variability on the structural safety of Nano-Engineered concrete and found that Mix design variability significantly changes the reliability index and failure probability. Adamu et al. (2024) also checked the reliability of Nano-Engineered RC beams and demonstrated the integration of material variability into Eurocode reliability assessment.

The performance of geopolymer concrete in terms of structural safety can be evaluated using probabilistic reliability analysis. In structural engineering, reliability is defined as the ability of a structural component to perform its intended function for a specified period under stated conditions without failure (Mohammed et al., 2014). Qianru and Ang (2013) stated that reliability-based analysis provides a rational means of accounting for uncertainties in material properties, geometry, and applied loads. The reliability of a structural member is commonly expressed in terms of its probability of failure or safety (reliability) index. The reliability of reinforced concrete members is also influenced by the adopted design provisions and the uncertainties associated with the constituent materials. García-Soto et al. (2017) opined that structural safety should be evaluated within a reliability framework to ensure that acceptable safety levels are maintained under different design conditions.

Based on the above statements, this study checks the effects of sodium hydroxide molarity, sodium silicate-to-sodium hydroxide ratio, and alkaline liquid-to-metakaolin ratio on the reliability of reinforced metakaolin-based geopolymer concrete beams using the First-Order Reliability Method (FORM). Developed compressive strength models developed from are incorporated into the failure modes for reliability analysis to evaluate the influence of material variability on the safety indices associated with flexural moment (In compression and Tension), shear, and deflection failure modes. The results from this study will be useful in identifying the failure mode that is most affected by the geopolymer materials and identifying the mix parameter that is most sensitive to the failure modes and hence, the structural reliability and safety of reinforced metakaolin-based geopolymer concrete beams.

2.0 MATERIAL AND METHODS

2.1 Materials:

The material use for the research were Metakaolin (Processed from kaolin calcination at 800°C for 3 h, based on Moodi *et al.* (2011). The fine and coarse aggregates, NaOH and Na₂SiO₃ were sourced locally. Potable water available in the heavy-duty laboratory of the Civil Engineering Department, A.B.U, Zaria, was used.

2.2 Methods

2.2.1: Concrete Production: The proportioning of the mix constituents is mostly done by weight, as captured in Mustafa *et al.* (2011) and Sameer (2015). The fine and coarse aggregate ratio used was 1:2. The cement (metakaolin-based geopolymer cement) content used in the mix was based on the average 2400kg/m³ concrete density. The NaOH Molar concentration, the ratio of Na₂SiO₃ to NaOH, and that of metakaolin to alkaline liquid were respectively 8 M, 3, and 0.9. The mixing and casting were in accordance with BS EN 12390-1 (2000) and BS EN 12390-2 (2000).

2.2.2: Production of Beam Specimen: Metakaolin-based geopolymer concrete beams reinforced with two (2) 8 mm diameter bars as tension reinforcement were produced. The beams had dimensions of 100 × 100 × 450 mm (short beam, because L/H <5 and shear span-to-depth ratio of 1 to 2.5).

2.2.3: Beam Test: The test was conducted per BS EN 12390 -5 (2009) (Plate I). Three (3) beams were produced with the same mix batch and reinforcement details were loaded to failure. The average failure load computed for the three beams was 39 kN.



Plate I: Centre Point Test of the Beam (3-point Bending Test)
The beam shown in Plate 1 was loaded at the centre point with rollers as supports

2.2.4 Compressive Strength Models: The models presented in equations (1) and (2) were developed by Aliyu (2021) using the Datafit statistical package (2019) to predict the compressive strength of the metakaolin-based geopolymer reinforced concrete beams.

$$f_{ck1} = -109.69 + 18.9Mc + 48.01ALr - 0.7163Mo^2 - 6.41ALr^2 - 1.98Mo * ALr \quad (1)$$

$$f_{ck2} = 80.109(ALm^{3.553}) \times 0.991^{Wc} \quad (2)$$

2.2.5 Probabilistic Reliability Analysis

Structural reliability according to Bai *et al.* (2018) is defined by:

$$R_r = 1 - P_f \quad (3)$$

Where P_f is the probability of failure. For the reliability (Probabilistic) analysis, the performance function or limit state function ($G(x)$) is defined by $G(x) = R - S$

2.2.5.1 Limit state functions

a) Performance Function for Moment Capacity

$$G_{mx} = M_r - M_a \quad (4)$$

$$\text{where } M_a = \theta_s(0.25pl + 0.125\gamma_c b h g l^2 \times 10^{-9}) \quad (5)$$

Equation (5) is the applied moment as a function of the applied point load and the beam's self-weight. For the probabilistic analysis, the partial factors of safety for steel (1.15) and concrete strength (1.5) recommended by EC2 were removed, and the moment capacity equations (based on compressive and tensile capacity in EC 2) were re-derived to yield;

i. MOMENT OF RESISTANCE OF THE CONCRETE COMPRESSIVE STRUT.

$$M_r = 0.251\theta_R f_c b d^2 \quad (6)$$

ii. TENSILE MOMENT CAPACITY (MOMENT OF RESISTANCE)

$$M_r = \theta_R f_{yk} \rho b d \left(d - \frac{f_{yk} \rho b d}{1.7 f_c b} \right) \quad (7)$$

b) Performance Function for Shear Capacity

$$G_{vx} = V_c - V_a = \theta_R \left(0.18 \left(1 + \frac{\sqrt{200}}{d} \right) (\rho f_c)^{1/3} \right) b d - \theta_s (0.5p + 0.5\gamma_c b h l g \times 10^{-9}) \quad (8)$$

Where M_r and M_a are the moments of resistance and applied moments, and V_c and V_a are the shear capacity and applied shear, respectively. $f_c, f_{yk}, p, g, \gamma_c, b, \rho, \text{ and } \rho'$ are the concrete strength, steel yield strength, applied load, acceleration due to gravity, unit weight of concrete, beam width,

tensile reinforcement ratio, and compression steel reinforcement ratio, respectively. θ_R is the model uncertainty.

2.2.5.2 Form Process. FORM5 is a reliability analysis software developed based on the manual presented by Gollwitzer et al. (1988). It was used for the probabilistic analysis.

2.2.5.3 Stochastic (Statistical) Parameters

Table 1: Stochastic Table of Variables

S/No	Variable	Unit	Mean	COV	Distribution Type
1	Unit Weight of concrete (Y_c)	Kg/m ³	2400	0.01	Normal
2	Breath (B)	mm	100	0.002	Normal
3	Depth (D)	mm	100	0.003	Normal
4	Length (L)	mm	450	0.0006	Normal
5	Steel yield Stress (f_{yk})	N/mm ²	397.3	0.15	Normal
6	Compressive strength (f_{ck})	N/mm ²	32.8	0.09	Normal
7	Reinforcement ratio (ρ)	Nil	1.24	0.17	Normal
8	Live Load (Q_k)	kN	39	0.35	Gumbel
9	Molar Concentration (Mc)	Mol/L	varied	0.05	Normal
10	Alkaline Liquid ratio (ALr)	---	varied	0.05	Normal
11	Alkaline Liquid ratio Metakaolin (ALm)	---	varied	0.05	Normal
12	Resistance Model Uncertainty (θ_R)	---	1	0.07	Lognormal
13	Load Model Uncertainty (θ_s)	---	1	0.2	Lognormal

Source: Aliyu (2021), Adamu et al. (2014), Abejide (2014), and Mohammed et al. (2014)

It should be noted that the COV and distribution types for Mc, ALr, and ALm were assumed to be 0.05 based on 95% confidence in deviation from target value and because Lane (2020) reported that many natural phenomena are at least approximately normally distributed.

3.0 RESULTS AND DISCUSSIONS

3.1 Compressive Strength Predictive Models

The plots (Figures 1a and 1b) present the experimental values used as input data to develop the models and the predicted values by the developed models. In other words, the plots present the experimental compressive strength values and the predicted compressive strength values by Equations (1) and (2), respectively. The statistical package (Datafit) reported a coefficient of multiple determination (R^2) of 0.634 for eqn (1) (Proportion of variance explained = 63.4%), meaning that, based on eqn (1), ALr and Mc have explained 63.4% of the variation in the MBGC compressive strength, while eqn (2) has an R^2 value of 0.932 (Proportion of variance explained = 93.2%) for Alkaline liquid ratio metakaolin content (ALm) and water content (Wc). When using the R^2 approach, the unexplained proportion of the variation, based on Andrew (2017), is attributed to other factors not measurable by the researcher or not considered in the model. The author further noted that many models developed by researchers with small R^2 values produce useful predictions. It can be said that, although the model captures a substantial portion of the behaviour, the unexplained 36.6 % variance might propagate into the safety computation. This cannot be avoided except if a model explains 100% of the variation.

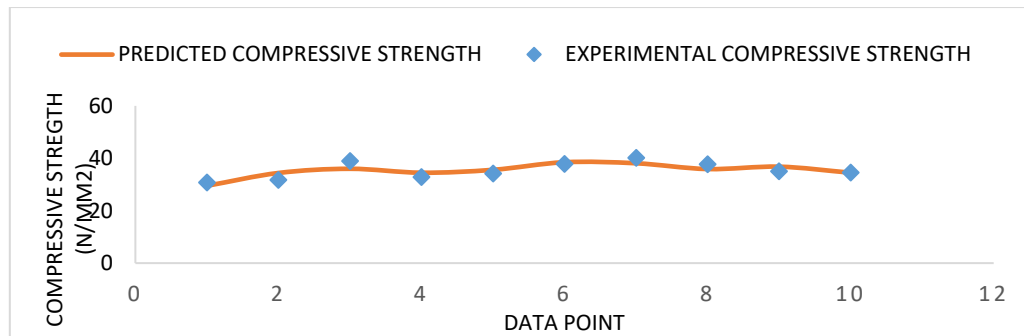


Figure 1a: Experimental and Predicted Data Points for equation 1 ($f_{cl}=f(Mc \text{ \& } ALr)$)

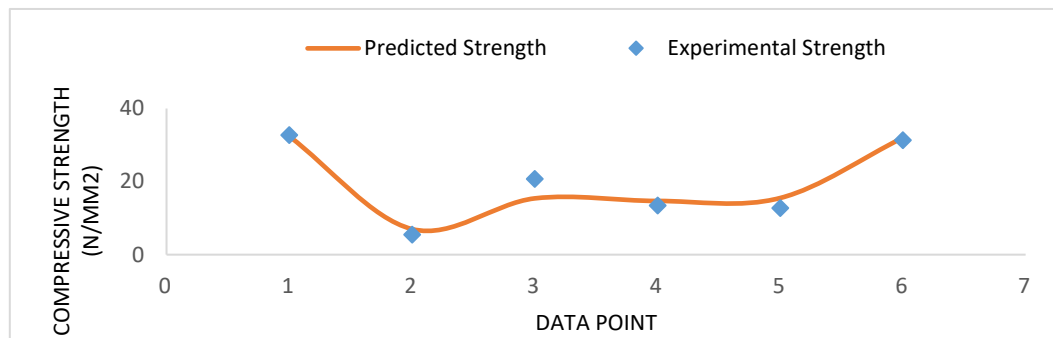


Figure 1b: Experimental and Predicted Data Points for equation 2 ($f_{cl}=f(ALm \text{ \& } Wc)$)

3.2 Effect of Variations in Mc and ALr on the Safety Index

The effects of Mc , ALr , and ALm variations on the safety index are presented in Figures 2, 3, 4, and 5, respectively. Each parameter affects the safety index of beam failure modes positively or negatively. An increase in molar concentration (Mc) from 6M to 10M (Figure 2) increases the safety index but from 10M to 16M a decrease was recorded; this is because, for the ALm (0.9) used in the mix, between 6M and 10M, the geopolymer gets increasing ionic species necessary for geopolymerisation and hence higher strength and safer structure, but beyond 10M the alkalinity (Ph or Mo of $NaOH$) of the solution might have been non-conductive for the reaction as reported by Alonso and Palomo (2001), therefore, the safety index drops as a consequence of lower strength of the concrete. When ALr was increased from 0.5 to 2.5 at 8M (Figure 3), the safety index increased since the geopolymerisation reaction is enhanced in the presence of more Na_2SiO_3 .

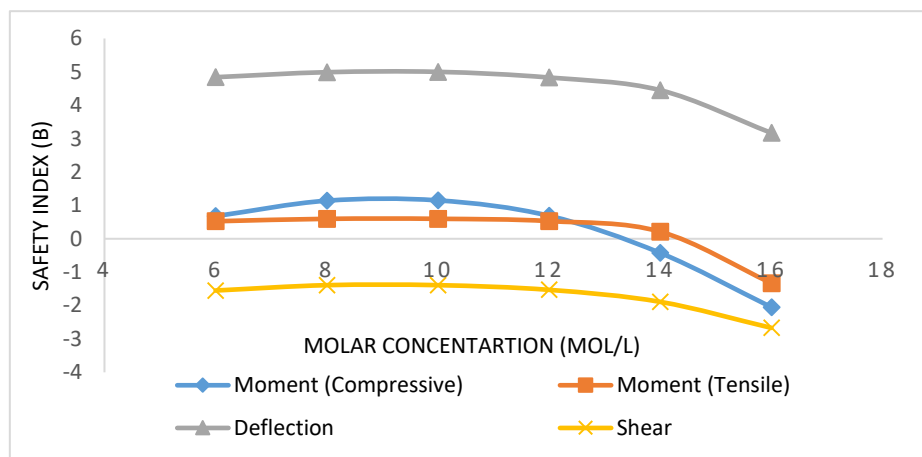


Figure 2: Molar Concentration (Mc) Effect on the Safety Index

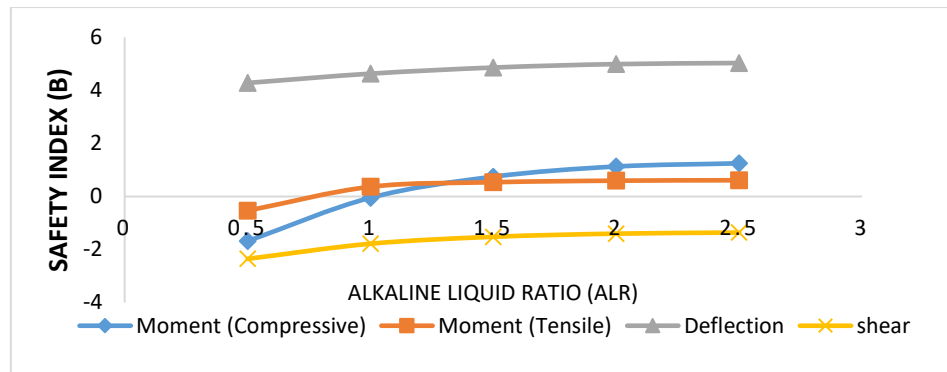


Figure 3: Alkaline Liquid ratio (Na_2SiO_3 : NaOH or ALr) effect on the safety index

3.3 Effect of Variations in ALm on the Safety Index

The factor that has the most significant effect on the safety index is the alkaline liquid ratio Metakaolin (ALm) (Figure 4). The safety index values increase distinctly with an increase in ALm; this might be for the reason that, with lower ALm ratios, the mix does not have sufficient alkaline liquid (AL) to break down the Al and Si precursors, but with an increase in AL, more of the Al and Si precursors react, forming more geopolymer cement and achieving better concrete compressive strength. The moment (flexural) failure mode is the most affected, followed by the shear failure mode, and the least affected is the limiting deflection.

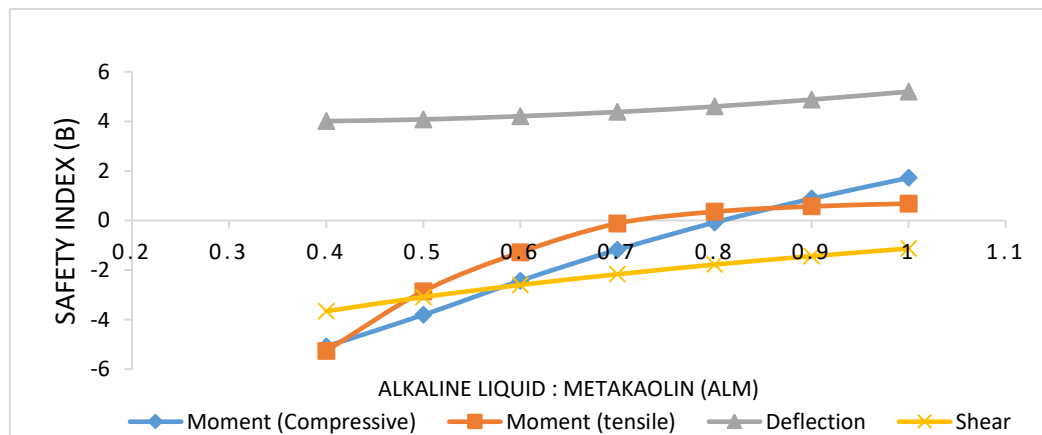


Figure 4: Alkaline Liquid: Metakaolin (ALm) Effect on the Safety Index

In summary, for all failure modes, an increase in M_c beyond 10M drops the safety index, while an increase in ALr up to 2.5 increases the safety index; Furthermore, an increase in ALm leads to a notable safety index increase.

Generally, the study also demonstrated that material variability significantly changes reliability index and failure probability, in agreement with Adamu et al. (2024).

4.0 CONCLUSIONS AND RECOMMENDATIONS

4.1 Conclusions

The conclusions derived from the study are;

1. The developed compressive strength models explained variation in compressive strength by 63.4% ($R^2 = 0.634$) for the M_c/ALr model, and by 93.2% ($R^2 = 0.932$) for the ALm/W_c model. This showed that the alkaline liquid-to-metakaolin ratio (ALm) is the primary variable controlling the compressive strength.
2. The mix composition that has the greatest influence on the safety/reliability index of metakaolin-based geopolymer reinforced concrete beam is the alkaline liquid-to-metakaolin ratio (ALm). Among the failure modes considered, the flexural (moment) failure was the most sensitive to the mix parameters.

3. Increasing the alkaline liquid ratio (ALr) up to 2.5 and molar concentration (Mc) from 6 M to 10 M improved the safety index. However, further increase beyond 10 M reduced structural safety.

It is recommended that, for safety reasons, the production of metakaolin-based geopolymer concrete intended for structural members should place greater emphasis on the metakaolin-to-alkaline liquid ratio, as this parameter has the greatest influence on the safety and structural performance of the member.

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