
DEVELOPMENT AND VALIDATION OF A FINITE ELEMENT MODEL FOR REINFORCED CONCRETE BEAMS EXTERNALLY STRENGTHENED IN FLEXURE WITH STEEL PLATES

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ABSTRACT

This study presented and validated a finite element (FE) model for estimating the structural performance of externally reinforced concrete (RC) beams using bonded steel plates. The developed model was validated with the existing experimental results, and the effect of the thickness of steel plate and the thickness of adhesive on the ultimate load of strengthened beams was studied. The FE predictions were in good agreement with the experimental results with differences in the ultimate loads being rather minor and the predictions being consistently higher. The developed model was accurate and reliable as indicated by a strong positive correlation (Pearson correlation coefficient, $r = 0.961$, $p < 0.001$). The ultimate load was used as a response variable and a two-factor analysis of variance (ANOVA) was conducted. The results showed that the thickness of the adhesive had a statistically significant effect on ultimate load ($F = 5.12$, $p = 0.043$) while steel plate thickness did not ($F = 2.12$, $p = 0.201$). The load–deflection responses also showed that the RC beams strengthened externally were considerably higher when compared to the control RC beam as a result of the application of external steel plate strengthening, in terms of stiffness, energy absorption capacity and ductility. The behaviour of the beams with an adhesive thickness ranging from 4 to 6 mm was optimal, showing high stiffness, energy absorption capacity, and high ductility due to the efficient dissemination of the stress on the steel–concrete interface. However, an increased thickness of the adhesive layer (8 mm) in comparison led to a decrease in structural efficiency due to a decrease in the interface stiffness and stress transfer. The study showed that finite element model could be used as a valid tool to predict the behaviour of steel-plated RC beams and that the effect of optimizing the thickness of the adhesive was more significant than the increasing thickness of the steel plates on the RC beam to attain better structural performance.

Keywords: Strengthening; Load; Numerical; Beam; Stiffness; Ductility; Energy absorption

INTRODUCTION

Reinforced concrete (RC) structures are a large share of civil engineering infrastructures around the world, known for their strength, durability, versatility, and cost-effectiveness (Tiza et al., 2024; Rusnak, 2025; Özmen & Tanrıverdi, 2025). During its service life, however, many existing RC structures suffer from deterioration or structural deficiency issues due to aging, reinforcement corrosion, increased service loads, design errors, environmental exposure, inadequate construction, seismic activities, accidental impacts, and changes in use of buildings (Yadeta et al., 2023; Rincon et al., 2024; Ma et al., 2025). These conditions can cause a decrease in the ability of structural members to withstand flexural loads, especially beams which are the main load resisting members (Mahi & Ridoy, 2025; Osta et al., 2026). Strengthening and rehabilitation, which are methods for prolonging the life of the structures without the high costs of demolition and reconstruction, have therefore become more and more important in recent years (Hu et al., 2024; Rusnak, 2025; Elkhatab et al., 2025).

Out of all the strengthening techniques that are available, externally bonded steel plate reinforcement is the most common technique used due to the fact that it is the most effective, easy to install and economic (Khalil & Attia, 2025; Mankare, & TS, 2026; Alsuhaibani, 2025). The method typically involves the use of high-strength epoxy adhesives to adhere steel plates to the tension surface of RC beams, and sometimes with mechanical anchors (Xu et al., 2024; Napoli et al., 2024). The flexural strength, stiffness, and cracking resistance of the composite structure are enhanced due to the external

bond of steel plate, and the deflection under service loads is reduced (Qiang et al., 2023; Mosahebfard et al., 2026). There are many experimental studies that have proven that steel plate strengthening is a viable solution for the rehabilitation of deficient RC beams and improves their structural performance (Bandara et al., 2023; Napoli et al., 2024; Xu et al., 2024; Elkhatib et al., 2025).

Although it has proven effective, the behaviour of externally strengthened RC beams is complex because of the interaction between the concrete, the reinforcing steel, the adhesive layers and the externally bonded steel plates (Foraboschi, 2022; Hamoda et al., 2024; Ahmed, 2025). The mechanical properties of the component materials, nature of the bonds between the concrete and the adhesive, geometric properties of the plates, configuration of the plates' strengthening, loadings and possible failure mechanisms such as crushing of the concrete, yielding of the steel, debonding of the plates or failure of the adhesive (Ali et al., 2001; Edalati & Irani, 2012; Alam et al., 2020). The prediction of the structural response is difficult with conventional analytical approaches only (Hamoush & Ahmad, 1990; Ciampa & Pecce, 2021).

Experimental tests are the most accurate way to study the behaviour of strengthened RC beams (Sun et al., 2023; Zhang et al., 2023). Laboratory testing, however, is costly, time consuming, and is often restricted in its availability of testing facilities, materials and instrumentation (Oehlers & Seracino, 2004; Teng et al., 2002). In addition, experimental programmes typically examine only a few of the variables, and it is challenging to explore the effects of several design parameters in a comprehensive way (Hajirasouliha et al., 2012; Abaqus, 2014). In view of this, numerical modelling, specifically Finite Element Method (FEM) has become an essential tool to foresee the response of RC structures under strengthening with externally bonded steel plates (Alharthai et al., 2024).

By using the Finite Element Method, complex material behavior can be simulated, the non-linear cracking, crushing of concrete, yielding of reinforcement steel, the behavior of the adhesive bond, and the interaction between steel plates and concrete can be modeled (Muka, 2023; Noor et al., 2025). The modern finite element software enables researchers to accurately study the distribution of stresses, the load-deflection behavior, propagation of cracks, development of strains, and failure modes (Kholil & Ahmed, 2025; Ciampa & Pecce, 2021). After being properly validated, finite element models are able to perform detailed parametric studies, optimize strengthening schemes and support structural design (Raviolo et al., 2023; Mobarhan et al., 2026).

Several researchers have created finite element models of RC beams externally strengthened with various modelling methods, constitutive material models, and bond-slip models (Ciampa & Pecce, 2021). In many of these studies, the results are in reasonable agreement with the experimental data, but significant differences between experimental data and the results in the studies have been found due to the various assumptions made about material nonlinearities, the modelling of the interfaces, the mesh discretization, and the failure criteria (Chen et al., 2012). Furthermore, existing models have predominantly considered strengthening with externally bonded carbon fibre reinforced polymer (CFRP) while few studies have investigated externally bonded steel plate strengthening in detail and developed validated nonlinear finite element models that can be used to accurately predict both the global and local structural response (Li et al., 2025).

To create a good finite element model which realistically simulates the behavior of exteriorly strengthened reinforced concrete beams is therefore imperative. A model, it should be capable of reproducing experimental observations such as load-deflection behaviour, strain distribution, cracking patterns, ultimate load capacity and failure modes (Li et al., 2025). It is important to validate the model developed against pre-existing experimental studies before it can be used with confidence for additional analysis and structural design applications, as proposed by Ciampa & Pecce (2021).

The focus of this study is thus the development and validation of the nonlinear finite element model of reinforced concrete beams strengthened externally in flexure with steel plates. To ensure the accuracy and reliability of the model, published validation test results of existing studies will be used. The validated model is hoped to be able to offer a reliable numerical tool in analysing the flexural response of strengthened RC beams and minimize the need for expensive experimental investigations thus helping in the development of structural rehabilitation and strengthening.

2.0 METHODOLOGY

2.1 Development of Finite Element Model of Steel Glue RC Beam

This section describes the formulation of a finite element model that is based on the ideas originally presented by Bimal & Hiroshi (2002). Considering the very small thickness of the interface, an eight-noded element as shown in Figure 1(a) was found to be sufficient for modeling the steel-glue-concrete interface.

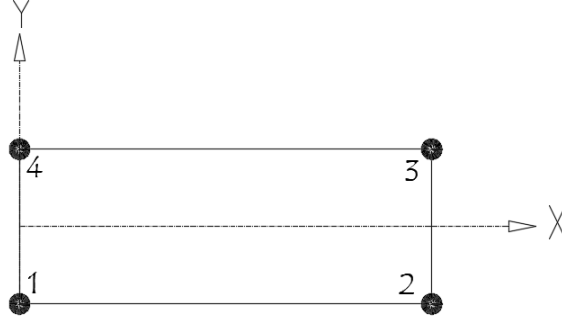


Figure 1: Steel-glue concrete interface element

Introducing pseudo-nodes 1' and 2' (see Figure 1), an iso-parametric formulation with parabolic shape functions could be implemented. The element was formulated using the two-point Gaussian integration rule.

2.1.1 Stress- strain relations

The strains in the glue element can be written as

$$\begin{aligned} \varepsilon &= \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} \\ &= \begin{bmatrix} \frac{\delta u}{\delta x} \\ \frac{\delta v}{\delta y} \\ \frac{\delta u}{\delta y} + \frac{\delta v}{\delta x} \end{bmatrix} \end{aligned} \quad (1)$$

The strain ε_x along the x -direction may be approximated as

$$\begin{aligned} \varepsilon_x &= \frac{\delta u}{\delta x} \end{aligned} \quad (2)$$

A shape function is an interpolation function that provides an association between the variables in a finite element and their values at the nodes of the element. The finite element methods are applied to solve the problem and these nodal values are obtained.

$$\begin{aligned} u(x) &= \sum_i N_i(x)u_i \\ &= \mathbf{N}u \end{aligned} \quad (3)$$

Where;

$u(x)$ is the function under investigation (for example displacement field)

$\mathbf{N}(x)$ is the shape functions matrix

$$u = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ \vdots \\ u_n \end{bmatrix} \text{ is the vector of unknowns in the nodes (for example: displacement)}$$

Substituting Equation 3 into 2, we have,

$$\begin{aligned} \varepsilon_x &= \frac{\delta u}{\delta x} = \frac{\delta}{\delta x} \sum_{i=1}^2 N_i'(x) \check{u}_i \\ &= \sum_{i=1}^2 \frac{dN_i'(x)}{dx} \check{u}_i, \end{aligned} \quad (4)$$

The shape functions at three equivalent nodal points 1' and 2' as shown in Fig.1 are represented by $N_i = N_i(x)$; u_i are the average nodal displacements along the line $n = 0$. This equation can then be written in terms of the nodal displacements of the element, u_i , as shown in equation (4).

$$\begin{aligned} \varepsilon_x &= \frac{dN_1'}{dx} \left(\frac{u_1 + u_4}{2} \right) \\ &\quad + \frac{dN_2'}{dx} \left(\frac{u_2 + u_3}{2} \right) \end{aligned} \quad (5)$$

The strain in the normal direction ε_n is given by

$$\begin{aligned} \varepsilon_y &= \frac{\delta v}{\delta y} \end{aligned} \quad (6)$$

and can be stated in terms of nodal normal strains as

$$\begin{aligned} \varepsilon_y &= \sum_{i=1}^2 N_i'(x) \left(\frac{\delta v}{\delta y} \right)_i = \sum_{i=1}^2 N_i'(x) \left(\frac{\Delta v}{t} \right)_i \\ &= \frac{1}{t} [N_1'(v_4 - v_1) + N_2'(v_3 - v_2)] \end{aligned} \quad (7)$$

where t is the element thickness. The shear strain γ_{xy} is approximated as

$$\begin{aligned} \gamma_{xy} &= \frac{\delta u}{\delta y} + \frac{\delta v}{\delta x} = \sum_{i=1}^2 N_i'(x) \left(\frac{\delta u}{\delta y} \right)_i + \sum_{i=1}^2 \frac{dN_i'(x)}{dx} \check{v}_i \\ \gamma_{xy} &= \frac{1}{t} [N_1'(u_4 - u_1) + N_2'(u_3 - u_2)] \\ &\quad + \left[\frac{dN_1'}{dx} \left(\frac{v_1 + v_4}{2} \right) + \frac{dN_2'}{dx} \left(\frac{v_2 + v_3}{2} \right) \right] \end{aligned} \quad (8)$$

Substituting Equations (5), (7) and (8) in Equation (1) we obtain, the above equation can be written as

$$\begin{aligned} \varepsilon &= [B][v] \end{aligned} \quad (9)$$

$$\varepsilon = \begin{bmatrix} \frac{1}{2} \frac{dN_1'}{dx} & 0 & \frac{1}{2} \frac{dN_2'}{dx} & 0 & \frac{1}{2} \frac{dN_2'}{dx} & 0 & \frac{1}{2} \frac{dN_1'}{dx} & 0 \\ 0 & -\frac{N_1'}{t} & 0 & -\frac{N_2'}{t} & 0 & \frac{N_2'}{t} & 0 & \frac{N_1'}{t} \\ -\frac{N_1'}{t} & \frac{1}{2} \frac{dN_1'}{dx} & -\frac{N_2'}{t} & \frac{1}{2} \frac{dN_2'}{dx} & \frac{N_2'}{t} & \frac{1}{2} \frac{dN_2'}{dx} & \frac{N_1'}{t} & \frac{1}{2} \frac{dN_1'}{dx} \end{bmatrix} \begin{bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \\ u_3 \\ v_3 \\ u_4 \\ v_4 \end{bmatrix}$$

Where:

B is the strain matrix and v is the displacement vector

N_i are the parabolic shape functions

$$B = \begin{bmatrix} \frac{1}{2} \frac{dN_1'}{dx} & 0 & \frac{1}{2} \frac{dN_2'}{dx} & 0 & \frac{1}{2} \frac{dN_2'}{dx} & 0 & \frac{1}{2} \frac{dN_1'}{dx} & 0 \\ 0 & -\frac{N_1'}{t} & 0 & -\frac{N_2'}{t} & 0 & \frac{N_2'}{t} & 0 & \frac{N_1'}{t} \\ -\frac{N_1'}{t} & \frac{1}{2} \frac{dN_1'}{dx} & -\frac{N_2'}{t} & \frac{1}{2} \frac{dN_2'}{dx} & \frac{N_2'}{t} & \frac{1}{2} \frac{dN_2'}{dx} & \frac{N_1'}{t} & \frac{1}{2} \frac{dN_1'}{dx} \end{bmatrix}$$

$$v = \begin{bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \\ u_3 \\ v_3 \\ u_4 \\ v_4 \end{bmatrix}$$

2.1.2 Shape functions

The shape functions can be defined as:

$$N_k = \sum \frac{\xi - \xi_m}{\xi_k - \xi_m}, \text{ therefore,}$$

$$N_1 = \frac{\xi - \xi_2}{\xi_1 - \xi_2} = \frac{\xi - 1}{-1 - 1}$$

$$N_1 = \frac{1}{2} (1 - \xi) \quad (12)$$

$$N_2 = \frac{\xi - \xi_1}{\xi_2 - \xi_1} = \frac{\xi + 1}{1 + 1}$$

$$N_2 = \frac{1}{2} (1 + \xi) \quad (13)$$

2.2 Material and Cracking Behavior of the Interface

Before cracking of the interface, the property matrix of the material is given as presented as Eq. (14), where: ν = Poisson's ratio of the interface, E = modulus of elasticity of the interface, G = shear modulus.

The behaviour of the interface will be assumed to be elastic under tensile and shear loading, with a linear response up to the failure limit as shown in Figure 2. After reaching this limit, cracking starts to develop in the interface element. The very thin interface limits crack propagation to along the adhesive (glue) line or perpendicular to the glue line.

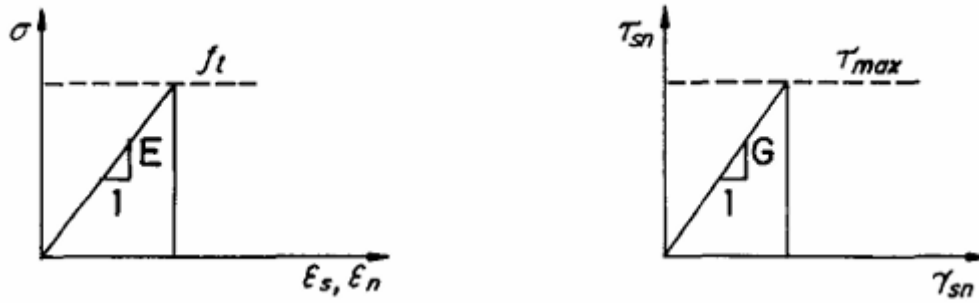


Figure 2: Material behavior of the interface element in tension and shear.

$$[D]_{x-y} = \frac{E}{1-\nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{G(1-\nu^2)}{E} \end{bmatrix} \quad (14)$$

It is assumed that cracking occurs perpendicular and/or parallel to the glue line when (fii) and/or (fip) equals the corresponding normal stress at the interface. From this point, further load in the direction of the crack can no longer be resisted by the interface. Therefore, the normal stress and the corresponding elastic modulus in that direction are assumed to be zero, i.e., total loss of stiffness across the crack.

If cracks form at an angle normal to direction of glue, the property matrix of the material should be modified to reflect the loss of stiffness in that direction and the reworked property matrix is called:

$$[D]_{i-n} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & E & 0 \\ 0 & 0 & 0.25G \end{bmatrix} \quad (15)$$

In addition to cracking perpendicular to the glue line, the interface can also fail parallel to the glue line when it is subjected to a combination of normal and shear stresses. Experimental results suggest that such a failure can be best represented by the Mohr–Coulomb fracture criterion, which takes into consideration the interaction between normal and shear stresses at the interface, and predicts the onset of cracking under combined loading.

$$\tau = c - \sigma \tan \phi \quad (16)$$

where: c = cohesion, σ = normal stress, ϕ = angle of internal friction. For steel–epoxy–concrete interface: Cohesion c = 4.8-9.5 MPa and Internal friction angle ϕ =28°

The strength of adhesion and failure resistance of the interface is influenced by the type of surface preparation, the quality of the adhesive, and properties of the concrete substrates. If the failure occurs at the interfaces due to tensile (peeling) stress, then the stress components and stiffness terms at the interfaces are set to zero, indicating that the bond will not carry any load. But when the loading is compressive, failure defined by the Mohr–Coulomb criterion enables the interface to sustain a compressive load. Here, only the shear stress and shear stiffness are removed, and the compressive normal stiffness is left as it is, and a new material stiffness matrix is obtained.

$$[D]_{x-y} = \begin{bmatrix} \frac{E}{1-\nu^2} & \frac{\nu E}{1-\nu^2} & 0 \\ \frac{\nu E}{1-\nu^2} & \frac{E}{1-\nu^2} & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (17)$$

2.2.1 Material Properties for the Interface Element

The following material properties define the interface element: normal elastic modulus (E), shear modulus (G), Poisson's ratio (ν) and tensile strength (f_{ti}). The properties of these are determined by those of the parent components, which are the concrete substrate, epoxy adhesive and the externally bonded steel plate. The mechanical properties of these constituent materials are used to determine the

normal elastic modulus, the Poisson's ratio and the tensile strength, and the shear modulus of the interface is obtained by measuring the slope of the average shear stress–slip relationship of a half beam specimen test.

The resulting shear modulus was found to be $G=120.1$ MPa. This is essentially the same as the value reported by Charif (1983). If $\nu = 0.16$, the interface modulus is calculated as $E=2G (1+\nu)$ and comes out to be 278.6 Mpa. The experimental value of the tensile cut-off strength of the interface was obtained as $f_{ti} = 4$ MPa.

2.2.2 Finite Element Model for Constituent Materials

The interface element of the previous section was added to the nonlinear finite element solution procedure. The response of each of the constituent materials needs to be accurately modelled when dealing with reinforced concrete beams strengthened by externally bonded steel plates (EBSSP), which have highly nonlinear behaviour. The overall structural response will be dependent on the interaction between the concrete, internal reinforcing steel, externally bonded steel plate and the bond interfaces. Accordingly, an appropriate constitutive model and finite element formulation were assigned to each material to realistically simulate its behaviour during the loading process.

2.3 Concrete

2.3.1 Yield Criterion

Cracking in tension and crushing in compression cause the response of concrete to be highly nonlinear. The Mohr–Coulomb yield criterion is used to describe the onset of yielding in this study. Initial yielding is assumed to take place at about 30% of the compressive strength of the concrete, expressed as:

$$\sigma_0 = 0.3f'_c \quad (18)$$

where f'_c is the compressive strength of the concrete. From this point on, the yield surface expands due to plastic deformation. The following loading surfaces are represented by

$$f(\sigma) = \sigma_0(\lambda) \quad (19)$$

The equation contains a hardening parameter, λ , that is a function of the effective plastic strain. During loading, the concrete displays elasto-plastic characteristics, and continual plastic deformation occurs. After the effective stress exceeds the ultimate compressive strength, the material is said to be perfectly plastic up to the crushing strain.

2.3.2 Flow Rule and Hardening Rule

For plastic deformation there is an associated flow rule, which is characterized by the plastic strain increment that is perpendicular to the yield surface. This relation between effective stress and plastic strain is obtained from uniaxial parabolic stress–strain relation proposed for concrete,

$$\sigma = E_0 \varepsilon - \frac{1}{2} \left(\frac{E_0}{\varepsilon_0} \right) \varepsilon^2 \quad (20)$$

where E_0 is the initial elastic modulus, ε is the total strain, and ε_0 is the strain at the peak compressive stress. The initial elastic modulus is computed according to the formula suggested by the ACI Committee 363,

$$E_0 = 3320\sqrt{f'_c} + 6900 \quad (21)$$

It applies to normal and high-strength concrete (between 21 and 83 MPa compressive strength). The constitutive relationship for concrete, then, can be written as

$$\sigma = -E_0 \varepsilon_p + \sqrt{2E_0^2 \varepsilon_0 \varepsilon_p}, \quad (0.3f'_c \leq \sigma \leq f'_c) \quad (22)$$

where ε_p denotes the effective plastic strain.

2.3.3 Crushing Criterion

It is assumed that crushing of concrete is not affected by applied stress, but rather by the strain level. The crushing is defined to happen when the compressive strain is effective enough to reach the ultimate crushing strain, as provided by the equation below

$$f(\varepsilon_{ij}) = \varepsilon_u \quad (23)$$

Where, $\varepsilon_u = 0.0035$. The concrete has been considered to have lost all its compressive load-bearing capacity at this point.

2.3.4 Cracking Behaviour

The tensile response of concrete is modeled by a smeared crack model to distribute cracking throughout the finite element instead of as a discrete crack. Concrete is a linear elastic material in tension before cracking. Cracks start when the maximum principal tensile stress equals the tensile strength of concrete. The tensile strength is estimated from the following empirical equation given by Raphael (1984),

$$f_t = 0.324(f'_c)^{2/3} \quad (24)$$

After cracking, it is assumed that cracks form at 90° to the maximum principal tensile stress. The perpendicular stiffness is set to zero, the shear stiffness is reduced to reflect the loss of aggregate interlock along a crack and Poisson's ratio for cracks is ignored.

2.3.5 Tension Stiffening

The concrete and the reinforcement act together. This means that concrete between cracks can still resist significant tensile force, although cracking reduces the effective tensile stiffness of the concrete significantly. This phenomenon is called tension stiffening and is represented in the finite element model by assigning an effective or fictitious post-cracking tensile modulus to concrete.

The equation can be written as:

$$E_i = \alpha f'_t \left(1 - \frac{\varepsilon_i}{\varepsilon_m}\right) \frac{1}{\varepsilon_i}, \quad \varepsilon_t \leq \varepsilon_i \leq \varepsilon_m \quad (25)$$

Where, E_i = s the secant (or degraded) modulus of elasticity at a tensile strain ε_i , α = is a material parameter that controls the post-cracking tensile response, f'_t = tensile strength of concrete, ε_i = tensile strain at the current loading stage, ε_t = strain at the onset of cracking, ε_m = maximum tensile strain.

2.4 Finite Element Representation

The concrete material is modeled as a finite element, with 9 integration points and 9 nodes (isoparametric elements) and the constitutive matrix of the model is changed at each integration point in each load increment to capture concrete cracking, crushing and plastic behaviour accurately. The internal reinforcing steel is idealized using 3-noded quadratic truss elements, which are only capable of handling axial deformations, and with an elasto-plastic constitutive model with isotropic hardening. This bond between the reinforcing steel and concrete is modeled with the use of six-noded interface elements allowing bond-slip and failure when the interface shear exceeds the given bond strength. The interface has a shear modulus of 1.2GPa, elastic modulus of 2.78GPa, Poisson's ratio of 0.16, and bond strength of 7.5MPa. The externally bonded steel plate is modeled as an elasto-plastic material with isotropic hardening based on the von Mises yield criterion and discretized by nine-noded isoparametric Lagrangian elements to capture the stress concentrations and peeling stresses. Taken together these material models allow for the finite element model to simulate realistically the concrete damage, the yielding of the steel, and the bond-slip and degradation of the interface, providing reliable prediction of the loading-deformation behavior, failure mode and ultimate load capacity of steel-plated reinforced concrete beams under monotonic loading.

2.5. Beams investigated

A custom MATLAB program based on an incremental–iterative Newton–Raphson solution procedure was developed and used to implement and solve the finite element model. The nonlinear equilibrium equations were solved incrementally by subdividing the applied load into a number of load steps. The global tangent stiffness matrix was assembled from individual element stiffness matrices and the incremental displacement vector was obtained by solving the global equilibrium equations for each load step. The iterative solution accounted for the nonlinear response of the concrete, the steel reinforcement, the external steel plate and the interface elements. At the end of each iteration the element strains and stresses at the integration points were calculated and the internal resisting force vector was modified. The residual force was then calculated as the difference between the externally applied. Both force and displacement criteria were used to converge each load increment. Convergence was assumed to be achieved when the normalized residual force and incremental displacement were small enough, with tolerance limits of and, respectively. A maximum of 50 iterations was allowed for each load increment. Where convergence was not achieved the load increment was reduced and the iterative procedure repeated until equilibrium was achieved.

The beam specimens investigated by John (2023) are shown in Table 1. For the purposes of this study, the calibration and validation of the finite element model presented used the steel-plate-strengthened beam subset of John’s (2023) experimental program. The specimens considered were the control beam and the beams strengthened externally with steel plates of different thicknesses and adhesive layer thicknesses. The CFRP-strengthened specimens of John’s (2023) experimental program were not part of the FE validation dataset.

The sample ID is defined as follows in Table 1: P = Steel Plate, F = Flexural Strengthening, 0, 2, 4, 6, and 8 are the thicknesses of the epoxy adhesive (in mm), respectively. The material properties used in the finite element model were taken from the experimental study done by John (2023). The cylinder compressive strength (f'_c) was assumed for the concrete material as 85% of the cube compressive strength as reported in the experimental investigation, which is a common assumption in concrete modelling. The rest of the parameters needed to define the concrete constitutive model were obtained as follows:

The internal reinforcing bars and stirrups were simulated as elastic–plastic materials with $E_s = 200$ GPa and $f_y = 530$ MPa, and $H' = 200$ MPa. In the same manner, the elastic modulus (E_p) of the externally bonded steel plate was assumed to be 200 GPa, the yield strength (f_y) as 240 MPa, the Poisson's ratio (ν) as 0.30, and the plastic modulus (H') as 200 MPa.

The bond behaviour at the steel plate–adhesive–concrete interface was simulated using some of the interface properties proposed by Ziraba and Baluch (1995) to enable the numerical model to realistically simulate the interaction between the bonded steel plate and the concrete substructure.

Table 1: Experimental Programme of Steel Plate for Flexural Strengthening

Sample group	Steel Plate thickness	Epoxy Thickness (mm)	Sample ID
A	1.0mm Steel Plate	-	FA-0
		2	PFA2
		4	PFA4
		6	PFA6
		8	PFA8
B	1.5mm Steel Plate	2	PFB2
		4	PFB4
		6	PFB6
		8	PFB8
		2	PFC2
C	2.0mm Steel Plate	4	PFC4
		6	PFC6
		8	PFC8

Figure 3 presents the experimental beam layout and the cross-sectional reinforcement details used as basis for the finite element idealization, including the 2Ø8 mm top reinforcement, 2Ø10 mm bottom reinforcement, Ø6 mm stirrups and the externally bonded strengthening system. The cross-sectional size of the reinforced concrete beams was 150 mm × 100 mm as shown in Figure 3. The beams were longitudinally reinforced with 2Ø8 mm bars at the top and 2Ø10 mm bars at the bottom. Ø6 mm stirrups were provided along the length of the beam. The internal reinforcement of the control beam FA-0 and the strengthened beam PFA2 was identical, and the strengthening of PFA2 was achieved by an externally bonded steel plate using the specified adhesive layer.

The longitudinal reinforcement was modelled by truss elements embedded into the concrete domain in the finite element model. The top and bottom reinforcement was configured to be the same as the physical reinforcement in Figure 3. The Ø6 mm transverse reinforcement was also modelled with truss elements in the corresponding locations and spacing. The adopted bond-slip model was used to model the bond between the reinforcement and the surrounding concrete and to capture the relative interaction between the reinforcement and concrete. For the strengthened specimens, the externally bonded steel plate was modelled using plate elements and the adhesive layer between the steel plate and concrete was modelled using the six-node interface elements described in Section 2.5. Consequently, the reinforcement and strengthening configurations applied to the FE model were consistent with the geometry and reinforcement detailing of the experimental specimens as shown in Figure 3.

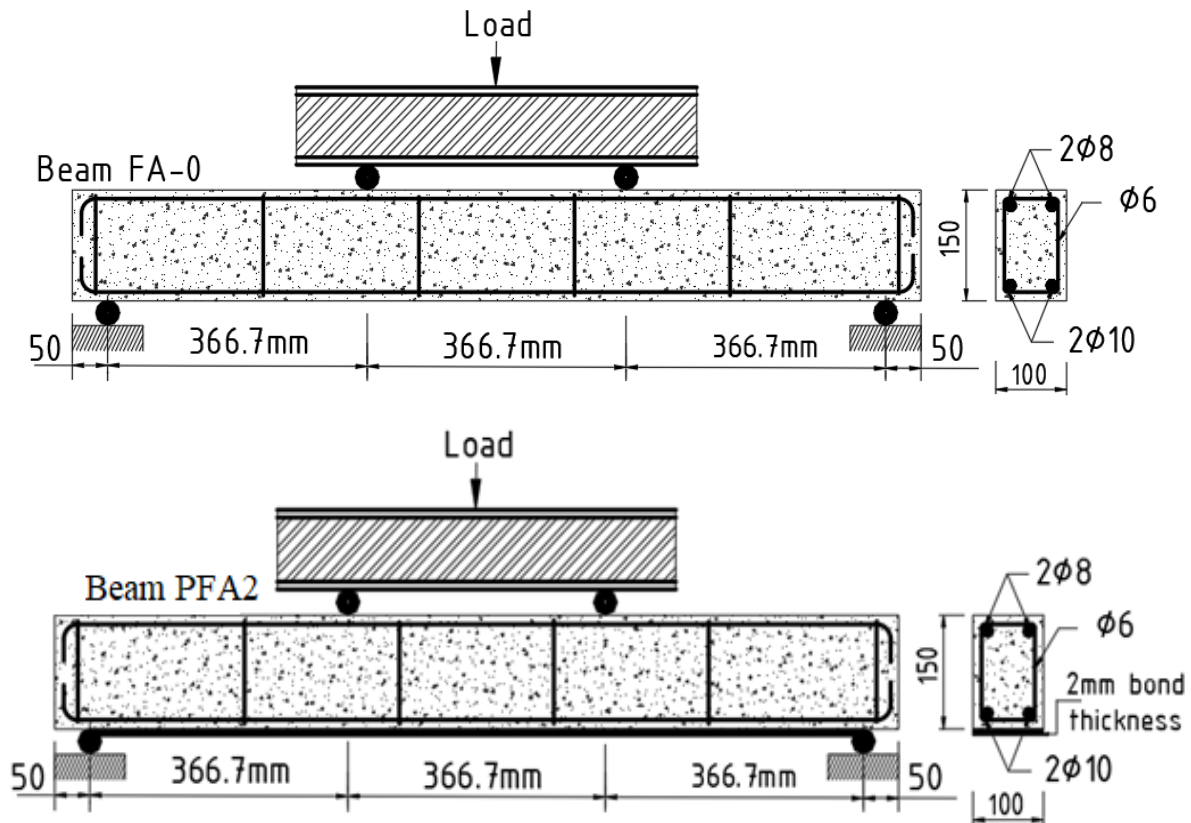


Figure 3: Beam configuration

2..5.1. Mesh sensitivity

A mesh sensitivity analysis was performed to find a suitable mesh density for the finite element model and to ensure that the numerical predictions were not significantly affected by the element size, as shown in Table 2. The three meshes that were studied are called coarse, medium and fine and are progressively refined. Similar element formulations, material constitutive models, interface properties, loading and boundary conditions were maintained for all mesh configurations. The predicted ultimate load and the corresponding mid-span displacement were the main parameters used to evaluate mesh convergence.

Table 2: Mesh sensitivity and convergence assessment

Mesh	Approx. element size (mm)	No. of elements
Coarse	40	4,820
Medium	30	7,650
Fine	20	12,480
Very fine	15	18,920

3.0 RESULTS AND DISCUSSION

3.1 Results and validation of the development model

The experimental results obtained by John (2023) were presented in Table 3, and the finite element (FE) results were compared with the experimental ones to check the agreement of the two methods and to assess the effect of steel plate thickness and adhesive thickness on the ultimate load values. The overall agreement between the FE model and experimental results was good with percentage differences ranging from 0.53% to 10.62% with an average difference of approximately 3.10%. The FE model always predicted the experimental ultimate loads slightly higher. Most of the specimens showed differences of less than 4%, which means a satisfactory numerical accuracy. The largest differences were for PFB8 (10.62 %) and PFB2 (6.82 %). This may be due to the difficulty in accurately modeling the adhesive thickness, interface behavior, debonding, surface imperfections and material variability. This was additionally verified by a high positive correlation between the experimental and FE results with a Pearson correlation coefficient of 0.961 ($p < 0.001$), showing that the developed FE model was able to capture the experimental behaviour of strengthened beams. In addition, a 2-factor analysis of variance (ANOVA) was performed with steel plate thickness and adhesive thickness as the independent factors and the FE-predicted ultimate load as the dependent variable, excluding the control specimen FA-0 as shown in Table 4. The ANOVA result indicates that the thickness of the steel plate is not statistically significant with the predicted ultimate load with value $F = 2.12$ and $p = 0.201 > 0.05$, while the thickness of the adhesive is statistically significant with the predicted ultimate load value $F = 5.12$ and $p = 0.043 < 0.05$.

Table 3: Loads of the Beams

Sample ID	Steel plate thickness (mm)	Adhesive thickness (mm)	John (2023)'s Exp. Results (kN)	Finite Element Model, (kN)
FA-0	-	-	93.33	94.24
PFA2	1.0	2.0	121.68	123.12
PFA4	1.0	4.0	128.23	130.10
PFA6	1.0	6.0	124.25	126.05
PFA8	1.0	8.0	115.75	119.23
PFB2	1.5	2.0	110.55	118.09
PFB4	1.5	4.0	119.08	122.09
PFB6	1.5	6.0	128.23	130.14
PFB8	1.5	8.0	105.00	116.15
PFC2	2.0	2.0	125.12	128.92
PFC4	2.0	4.0	127.24	130.95
PFC6	2.0	6.0	126.50	127.17
PFC8	2.0	8.0	115.00	119.4

Table 4: ANOVA parameters

Source	df	Sum of Squares	F-value	p-value
Steel plate thickness	2	50.55	2.12	0.201
Adhesive thickness	3	183.42	5.12	0.043
Error	6	71.62	—	—

These results show that the increase in the thickness of the adhesive layer compared to the variation in the thickness of the steel plates had a more significant effect on the load bearing capacity of the strengthened beams within the range of parameters investigated. The results of the analysis prove that the finite element model can represent the structural response of the steel-plated reinforced concrete beam well. It also shows that optimizing the thickness of the adhesive is more important than increasing the thickness of the steel plates in order to increase the load carrying capacity of the strengthened beams. The highest performance was achieved with the thickness of the adhesive of ~ 4-6 mm, and with the thickness of the adhesive increased to 8 mm, the structural efficiency was reduced.

3.2 Load–Deflection behaviour

The load-deflection curves in Figure 4 prove that the use of EBSP resulted in the significantly higher stiffness, energy absorption and ductility of the reinforced concrete beams as compared to the unstrengthened control beam (FA-0).

Referring to Figure 4, the strengthened beams have higher initial slopes than the control beam, which shows that the initial stiffness of the strengthened beams is higher. This indicates that the beams with plating took higher loads with lesser deflection in the elastic portion. The enhanced specimens tended to show highest stiffness for beams with 4 mm and 6 mm thickness of adhesive, which indicates the better transfer of stress between the steel plate and concrete. Contrastingly, beams with an 8 mm layer of adhesive had a small decrease in stiffness, indicating that, thicker adhesives allow more deformation before the stresses are adequately carried from one side to the other.

According to Figure 4, the increased area under the load–deflection curves for the strengthened beams compared to the control beam also showed a significant increase in the energy absorbing capacity of the strengthened beam. The stress distribution was more uniform and the propagation of cracks was slowed down and delayed by the combined effect of the steel plate and adhesive layer over the wall, leading to the increase in absorbed energy. The specimens with load application by 4 mm and 6 mm adhesive thickness demonstrated the maximum absorption of energy with a relatively high loading force and a significant deflection range prior to failure. On the other hand, the 8 mm adhesive specimens showed reduced energy absorption, with a high drop in load after the peak load, suggesting that interface degradation started earlier.

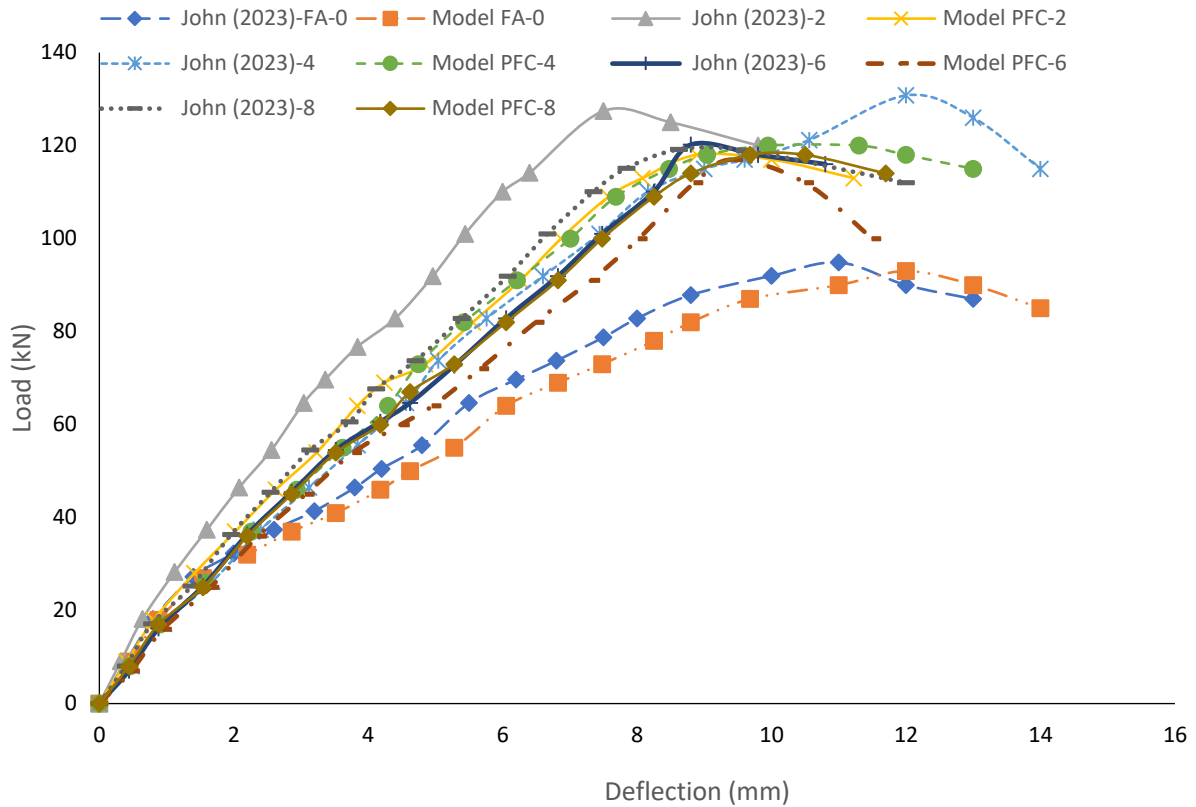


Figure 4: Load–Deflection behaviour

3.2.2 Ductility

The enhanced beams exhibited greater ductility than the control beam as they were able to carry higher deflections after yielding and still carried a significant percentage of their capacity. The post peak behavior of the plated beams was more continuous than that of the control beam thus reflecting a more ductile mode of failure. The specimens that had an adhesive thickness of 4 mm and 6 mm showed the best ductile behaviour, with high ultimate loads and long deformations until failure. Adhesive specimen dimensions of 8 mm reached relatively high loads, but the load at the peak was followed by a more evident decrease in load after the peak which indicates a loss in ductility due to loss of interface stiffness. As a whole, the results show that the optimum thickness range of the adhesive for the externally steel-plated reinforced concrete beams is 4–6 mm with regard to stiffness, energy absorption and ductility.

Table 5: Deflection and ductility Caparison

Beam	Exp. peak (mm)	Model peak (mm)	Difference (mm)	Difference (%)
FA-0	11.5	12.5	+1.0	+8.7
FA-2	8.0	9.5	+1.5	+18.8
FA-4	12.5	10.5	–2.0	–16.0
FA-6	9.5	9.5	0.0	0.0
FA-8	9.5	10.0	+0.5	+5.3

Table 5 presents deflection and ductility results of the experimental and FEM model. The best specimen in terms of large deformation capacity is FA-4, with the maximum experimental peak displacement of about 12.5 mm and a large deformation range. FA-6 and FA-8 show relatively stable deformation responses with peak experimental displacement of about 9.5 mm. FA-2, belonging to moderate-to-high ductility class, has the lowest experimental peak displacement of about 8.0 mm. FA-0 shows the baseline response with moderate ductility and

obvious post-peak softening. To summarize, FA-4 has the maximum deformation capacity, and FA-6 and FA-8 exhibit more stable post-peak behavior, which means that the FA modification influences the peak deformation and post-peak behavior of the beams.

3.3 Sustainability, Nigerian Infrastructure Development, Or Industrial Policy

The use of externally bonded steel plates for strengthening existing reinforced concrete structures has important implications for sustainable infrastructure development in Nigeria. Steel-plate strengthening is a practical, economical alternative to the demolition and complete reconstruction of structurally deficient beams and other reinforced concrete members. The technique can extend the functional service life and enhance the durability of aging infrastructure by increasing the load-carrying capacity, stiffness and serviceability of existing structures. This is especially true in Nigeria where the rehabilitation of dilapidated bridges, buildings and other reinforced concrete infrastructure is becoming increasingly important in the maintenance of safe and reliable infrastructure networks.

From a sustainability perspective the improvement of existing structures reduces the need for new construction materials, demolition activities and associated construction waste, and retains the embodied energy already invested in existing structures. The results of this study thus contribute to sustainable structural rehabilitation and infrastructure transformation by providing numerical and experimental evidence that can support the use of steel-plate strengthening as a viable rehabilitation technique. The approach is consistent with the conference themes on sustainability, Nigerian infrastructure development and industrial transformation by promoting resource-efficient rehabilitation, extending the service life of infrastructure and reducing the environmental and economic impact associated with premature demolition and reconstruction.

4.0 CONCLUSIONS

From the finite element analysis, validation with experimental results, and load–deflection behaviour analysis of the externally steel-plated reinforced concrete beams, the following conclusions are drawn:

1. The developed finite element model was found to be a good predictor of the structural response of externally steel-plated reinforced concrete beams. The numerical results were closely corresponded with the experimental results reported by John (2023), as the Pearson correlation coefficient (0.961) with p value < 0.001 , the developed model was reliable
2. The finite element model was always able to predict loads slightly higher than the experimental loads, but the differences were small and acceptable in engineering terms. This shows that the model can accurately predict the load carrying capacity of strengthened reinforced concrete beams.
3. Statistical analysis showed that the thickness of the adhesive significantly affected the ultimate load of the strengthened beams ($F = 5.12$, $p = 0.043$); however, the thickness of the steel plates in the range examined did not have a statistically significant effect on the ultimate load of the strengthened beams ($F = 2.12$, $p = 0.201$). This means that the impact of strengthening system is more related to the glue layer than to the thickness of the steel plate.
4. It was concluded that the structural performance of the reinforced concrete beams was significantly improved in terms of the stiffness, energy absorption potential and ductility by the exterior bonding of steel plates with the unstrengthened control beam. The enhanced beams performed better with respect to load resistance, post yield performance and failure gradient.
5. The overall performance of all the adhesive thicknesses tested was optimum for 4-6 mm. 4-6 mm was found to be the optimum thickness of the adhesive for optimum performance in terms of stiffness, energy absorbed and ductility.
6. Results showed that the thickness of 8 mm adhesive layer had no advantage for the structurally steel-plated reinforced concrete beam because of less interface stiffness and less effective stress transferring.

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